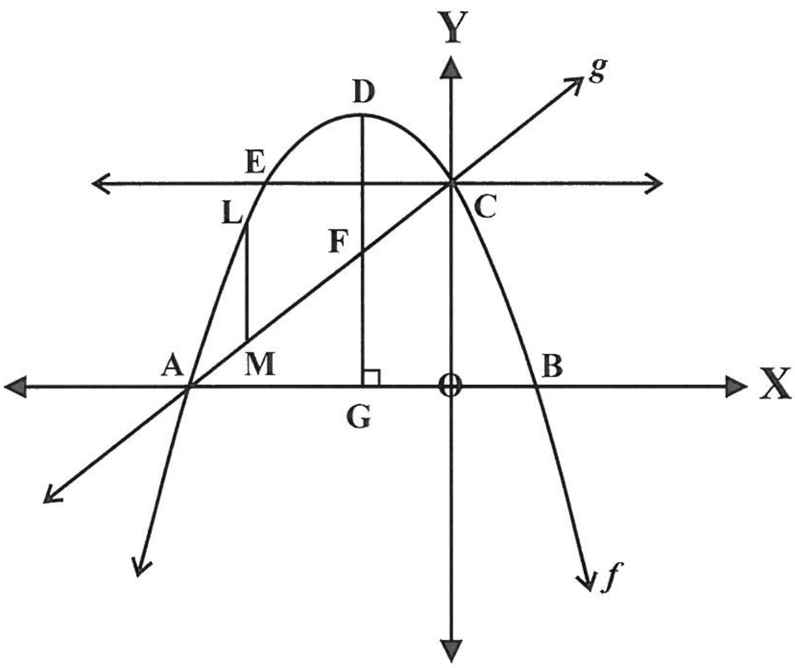
Grade 11 Test 1 Memo

60 minutes **Total: 50**

Question 1: [26]

In the sketch $f : x \rightarrow -(x+2)^2 + 9$ and $g : x \rightarrow mx + k$



1.1 Find the coordinates of A, B, C, D (the turning point of f) and E if EC // AB. (6)

1.2 Find the values of m and k. (3)

1.3 Find the average gradient of f between the x-values -3 and 0. (3)

1.4 Find x for which $f(x) < 0$. (2)

1.5 Determine the length of DF if DFG $\perp$ x-axis. (5)

1.6 Show that the length of LM $= -x^2 - 5x$, for L a point on f and M a point on g between the points A and C. LM extended, is $\perp$ on the x-axis. (3)

1.7 Determine the maximum length of LM. (4)

1.1. $f: x \rightarrow -(x+2)^2 + 9$ $\therefore D(-2;9)$ ✓

$y = -(x^2+4x+4)+9$

$y = -x^2-4x-4+9$

$y = -x^2-4x+5$ ✓ $\therefore C(0;5)$ ✓

For x-int: $(y=0)$

$\therefore 0 = -x^2-4x+5$

$0 = x^2+4x-5$

$0 = (x+5)(x-1)$

$x = -5$ or $x = 1$ $\therefore A(-5;0)$ ✓

and $B(1;0)$ ✓

For E: $y = 5 \Rightarrow C(0;5)$

$\therefore y = -x^2-4x+5$ 6

$5 = -x^2-4x+5$

$0 = x^2+4x$

$0 = x(x+4)$

$x=0$ or $x=-4$ $\therefore E(-4;5)$ ✓

1.2. $g(x)$ through $A(\overset{x}{-5};\overset{y}{0})$ and $C(0;5)$

$\therefore y = mx+k$ $\overset{\downarrow}{c}$

$y = mx+5$

$0 = m(-5)+5$ ✓

$5m = 5$ 3

$m=1$ ✓ and $k=5$ ✓

$\therefore g(x) = y = x+5$

1.3. If $x = -3$ then:

$y = -(-3)^2 - 4(-3) + 5$

$y = -(9) + 12 + 5$

$y = 8$

$\therefore$ AG through $\left(\overset{x_1}{-3}; \overset{y_1}{8}\right)$ and $\left(\overset{x_2}{\underline{\underline{0}}}; \overset{y_2}{5}\right) \Rightarrow C$

$\therefore AG = \dfrac{y_2 - y_1}{x_2 - x_1}$

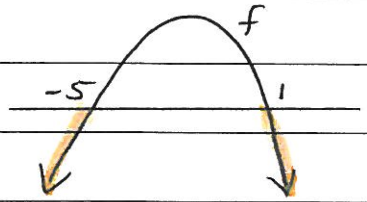
$= \dfrac{5-8}{0-(-3)}$

$= \dfrac{-3}{3}$ (3)

$AG = -1$

1.4. $f(x) < 0$

$\therefore x < -5$ or $x > 1$ (2)



1.5. $D(-2; 9)$ $\therefore DG = 9$

Substitute $x = -2$ in g

$\therefore g(-2) = -2 + 5$

$y = g(-2) = 3$ $\therefore FG = 3$

$\therefore DF = DG - FG$ (5)

$= 9 - 3$

$DF = 6$

1.6 $LM = f(x) - g(x)$

$= [-x^2 - 4x + 5] - [x + 5]$

$= -x^2 - 4x + 5 - x - 5$ 3

$LM = -x^2 - 5x$

1.7. $LM = -(x^2 + 5x)$

$= -\left[x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right]$

$= -\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right]$

$LM = -\left(x + \frac{5}{2}\right)^2 + \frac{25}{4}$

$\therefore$ Maximum length of LM is $\frac{25}{4}$

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Question 2: [12]

$$h(x) = 3^{x-1} + 2$$

2.1 Find the equation(s) of the asymptotes of h. (2)

2.2 Find the x- and y-intercepts of h. (4)

2.3 Sketch the graph of h. Show all intercepts and asymptotes clearly. (3)

2.4 Describe the transformation of $h \rightarrow p$ if $p(x) = h(-x)$. (2)

2.5 Write down the range of $h(x)$. (1)

2.1. Asymptote: $y = 2$ ✓ 2

2.2. x-int: $(y=0)$

$0 = 3^{x-1} + 2$

$-2 = 3^{x-1}$ ✓

No $\mathbb{R}$-solution ✓

$\therefore$ no x-intercept

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y-int: $(x=0)$

$\therefore y = 3^{0-1} + 2$ ✓

$y = 3^{-1} + 2$

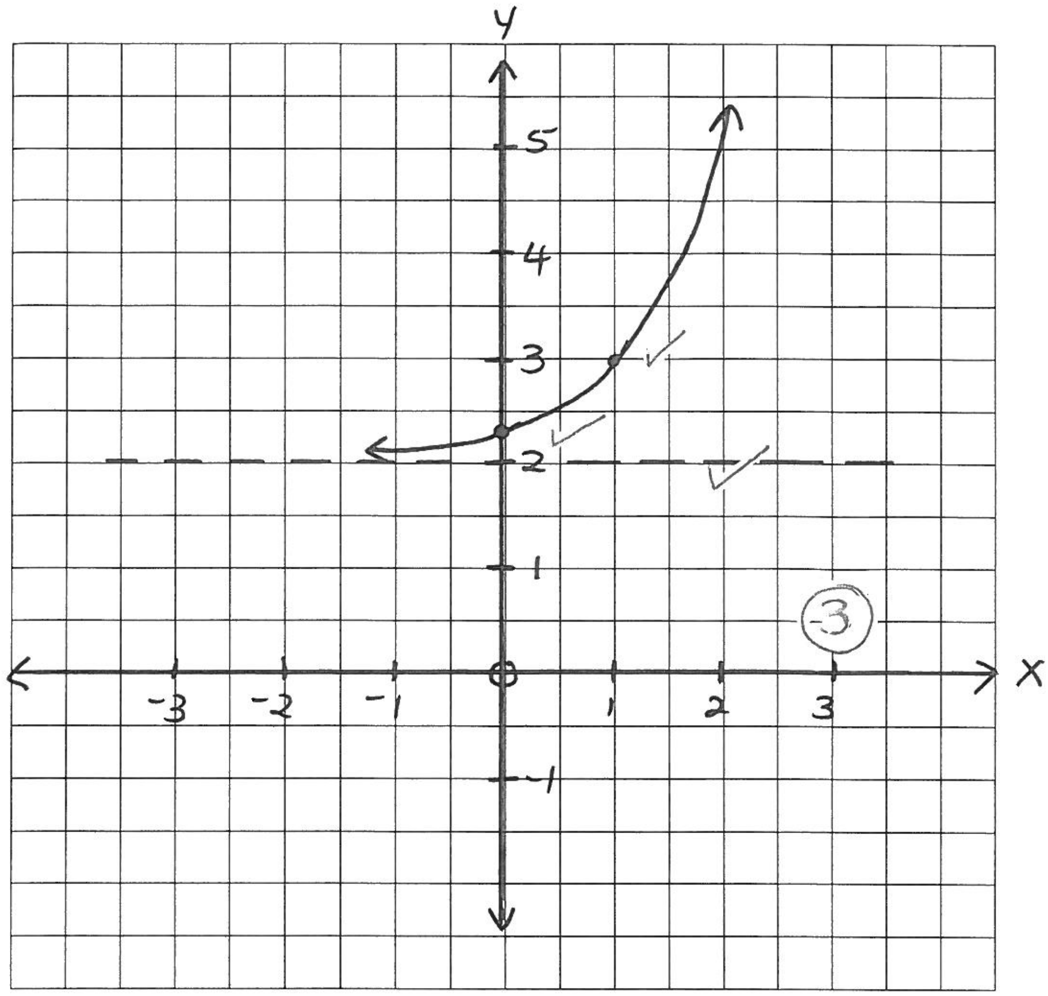
$y = \frac{1}{3} + 2$

$y = 2\frac{1}{3}$ ✓ $\therefore (0; 2\frac{1}{3})$

2.4 Reflection ✓ in the y-axis ✓ 2

2.5 R_h: $y > 2$ ✓ 1

2.3



Check point : $(x = 1)$

$$y = 3^{1-1} + 2$$
$$= 3^0 + 2$$
$$y = 1 + 2$$
$$y = 3$$

$\therefore (1;3)$

Question 3: [12]

$t(x) = \frac{-8}{x+p} + q$, with the equations of the asymptotes as $x = 1$ and $y = -2$.

3.1 Find the values of p and q. (2)

3.2 Calculate t(3). (2)

3.3 Find the coordinates of A and B, the intercepts of $t(x)$ on the axes. (6)

3.4 Find the equations of the positive symmetry axis of $t(x)$. (2)

3.1. $p = -1$ and $q = -2$ 2

3.2 $\therefore t(x) = \frac{-8}{x-1} - 2$

$\therefore t(3) = \frac{-8}{3-1} - 2$

$= \frac{-8}{2} - 2$

$= -4 - 2$ 2

$t(3) = -6$

3.3. x-int: $(y=0)$

$0 = \frac{-8}{x-1} - 2$

$2 = \frac{-8}{x-1}$

$2(x-1) = -8$

$2x - 2 = -8$

$2x = -6$

$x = -3$ $\therefore (-3;0)$

y-int: $(x=0)$

$y = \frac{-8}{0-1} - 2$

$y = \frac{-8}{-1} - 2$ 6

$y = 8 - 2$

$y = 6$ $\therefore (0;6)$

3.4. $y = x + k$ through $(\overset{x}{1}; \overset{y}{-2}) \Rightarrow$ intersection of asymptotes

$-2 = 1 + k$

$-3 = k$

$\therefore y = x - 3$ 2