Worksheet 2015-11 EG-00001(1)

(1) Determine whether A, B and C is collinear or not:

(a) A(1 ; 2) , B(3 ; 5) and C(5 ; 7)

A(1;2) and B(3;5)

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

$$= \frac{5-2}{3-1}$$

$$m_{AB} = \frac{3}{2}$$

B(3;5) and C(5;7)

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B}$$

$$= \frac{7-5}{5-3}$$

$$= \frac{2}{2}$$

$$m_{BC} = 1$$

∴ A, B and C is not collinear, because

$m_{AB} \neq m_{BC}$

(b) A(-1 ; 3) , B(4 ; 0) and C(14 ; 6)

A(-1;3) and B(4;0)

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

$$= \frac{0-3}{4-(-1)}$$

$$= \frac{-3}{4+1}$$

$$m_{AB} = \frac{-3}{5}$$

B(4;0) and C(14;6)

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B}$$

$$= \frac{6-0}{14-4}$$

$$= \frac{6}{10}$$

$$m_{BC} = \frac{3}{5}$$

∴ A, B and C is not collinear, because

$m_{AB} \neq m_{BC}$

(2) M(-2 ; -4) , N(1 ; -3) , R(2 ; -1) , T(-3 ; -1) and K(3 ; -4)

(a) Determine which of the following lines are parallel and which are perpendicular: MN , TK , RK , NR and TM

(b) Without calculating the angle of inclination, determine which of the lines in (a) have an acute angle of inclination.

(c) Calculate the angle of inclination of line TN.

(a) $m_{MN} = \frac{y_N - y_M}{x_N - x_M} = \frac{-3-(-4)}{1-(-2)} = \frac{-3+4}{1+2} = \frac{1}{3}$

$m_{TK} = \frac{y_K - y_T}{x_K - x_T} = \frac{-4-(-1)}{3-(-3)} = \frac{-4+1}{3+3} = \frac{-3}{6} = \frac{-1}{2}$

$m_{RK} = \frac{y_K - y_R}{x_K - x_R} = \frac{-4-(-1)}{3-2} = \frac{-4+1}{3-2} = \frac{-3}{1} = -3$

$m_{NR} = \frac{y_R - y_N}{x_R - x_N} = \frac{-1-(-3)}{2-(1)} = \frac{-1+3}{2-1} = \frac{2}{1} = 2$

$m_{TM} = \frac{y_M - y_T}{x_M - x_T} = \frac{-4-(-1)}{-2-(-3)} = \frac{-4+1}{-2+3} = \frac{-3}{1} = -3$

∴ Parallel lines: RK // TM

∴ Perpendicular: MN ⊥ RK or MN ⊥ TM or TK ⊥ NR

(b) Lines MN and NR (positive gradient)

(c) $m_{TN} = \frac{y_N - y_T}{x_N - x_T} = \frac{-3-(-1)}{1-(-3)} = \frac{-3+1}{1+3} = \frac{-2}{4} = \frac{-1}{2}$

∴ $\tan\theta = m_{TN}$

$\tan\theta = -\frac{1}{2}$

∴ $\theta = 180^\circ - 26,6^\circ$

∴ $\theta = 153,4^\circ$

(3) D(-3 ; -1) , E(0 ; -4) , F(-1 ; y) , G(x ; 3) and H(2 ; 2). Calculate the value of:

(a) x, if EG // DH

(b) y, if FH ⊥ DE

(a) $m_{EG} = \frac{y_G - y_E}{x_G - x_E} = \frac{3-(-4)}{x-(0)} = \frac{3+4}{x} = \frac{7}{x}$

$m_{DH} = \frac{y_H - y_D}{x_H - x_D} = \frac{2-(-1)}{2-(-3)} = \frac{2+1}{2+3} = \frac{3}{5}$

$\therefore \frac{7}{x} = \frac{3}{5}$ (Parallel lines $\Leftrightarrow m_1 = m_2$)

$\therefore 7 \times 5 = 3 \times x$

$3x = 35$

$\therefore x = \frac{35}{3} = 11\frac{2}{3}$

(b) $m_{FH} = \frac{y_H - y_F}{x_H - x_F} = \frac{2-(y)}{2-(-1)} = \frac{2-y}{2+1} = \frac{2-y}{3}$

$m_{DE} = \frac{y_E - y_D}{x_E - x_D} = \frac{-1-(-4)}{-3-0} = \frac{-1+4}{-3} = \frac{3}{-3} = -1$

$\therefore \frac{2-y}{3} \times -1 = -1$ (Perpendicular lines)

$\therefore \frac{2-y}{3} = 1$

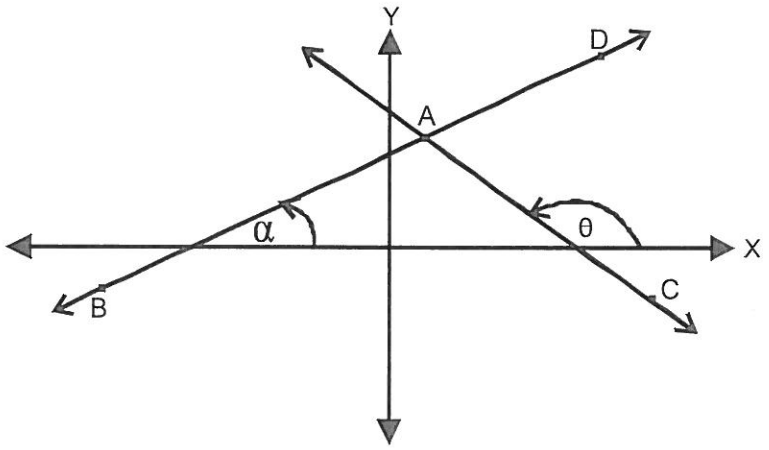
$2 - y = 1 \times 3$

$2 - y = 3$

$2 - 3 = y$

$\therefore y = -1$

(4) Calculate $\hat{DAC}$, correct to one decimal, with A(2 ; 5) , B(-6 ; -1) and C(7 ; -2):



$$m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{-2-5}{7-2} = \frac{-7}{5}$$

but $\tan\theta = m$

$\therefore \tan\theta = -\frac{7}{5}$

$\therefore \theta = 180^\circ - 54,5^\circ$

$\theta = 125,5^\circ$

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-1-5}{-6-2} = \frac{-6}{-8} = \frac{3}{4}$$

$\therefore \tan\alpha = \frac{3}{4}$

$\therefore \alpha = 36,9^\circ$

but $\theta = \alpha + B\hat{A}C$ [ext. $\angle$ of $\triangle$]

$\therefore 125,5^\circ = 36,9^\circ + B\hat{A}C$

$\therefore B\hat{A}C = 88,6^\circ$

but $B\hat{A}C + D\hat{A}C = 180^\circ$ [$\angle$s on str. line]

$\therefore D\hat{A}C = 91,4^\circ$