

(1) Simplify, without using a calculator: (Write your answer as a positive exponent!)

(a) $(3^1 x^3 y^{-2})^3$

$$= 3^3 x^9 y^{-6}$$

$$= \frac{27x^9}{y^6}$$

(b) $(9a^3)^{-1} (4a^3 b^{-2})^0$

$$= 9^{-1} a^{-3} \times 1$$

$$= \frac{1}{9a^3}$$

(c) $81^{\frac{1}{2}} \times x^{\frac{1}{4}} y^{\frac{1}{3}} \div (x^{\frac{1}{2}} y^{\frac{1}{3}})$

$$= (3^4)^{\frac{1}{2}} \times x^{\frac{1}{4} - \frac{1}{2}} \times y^{\frac{1}{3} - \frac{1}{3}}$$

$$= 3^2 \times x^{\frac{1}{4} - \frac{2}{4}} \times y^0$$

$$= 9 \times x^{-\frac{1}{4}} \times 1$$

$$= \frac{9}{x^{\frac{1}{4}}}$$

(d) $\frac{12^{x+1} \times 3^x}{9^{x-1} \times 4^x}$

$$= \frac{(2^2 \times 3)^{x+1} \times 3^x}{(3^2)^{x-1} \times (2^2)^x}$$

$$= \frac{2^{2x+2} \times 3^{x+1} \times 3^x}{3^{2x-2} \times 2^{2x}}$$

$$= 2^{2x+2-2x} \times 3^{x+1+x-(2x-2)}$$

$$= 2^2 \times 3^{x+1+x-2x+2}$$

$$= 2^2 \times 3^3$$

$$= 4 \times 27 = 108$$

$$\begin{aligned}
 \text{(e)} \quad & \frac{2^{m-1} + 2^m}{2^{m+1}} \\
 &= \frac{2^m \times 2^{-1} + 1 \times 2^m}{2^m \times 2^1} \\
 &= \frac{2^m \left(\frac{1}{2^1} + 1 \right)}{2^m \times 2^1} \\
 &= \frac{2^m \left(1\frac{1}{2} \right)}{2^m \times 2} \\
 &= \frac{\frac{3}{2}}{2} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & \frac{5^{\frac{1}{2}} \times 20^{\frac{3}{4}} \times 8^{\frac{5}{6}}}{5^{\frac{1}{4}}} \\
 &= \frac{5^{\frac{1}{2}} \times (2^2 \times 5^1)^{\frac{3}{4}} \times (2^3)^{\frac{5}{6}}}{5^{\frac{1}{4}}} \\
 &= \frac{5^{\frac{1}{2}} \times 2^{\frac{3}{2}} \times 5^{\frac{3}{4}} \times 2^{\frac{5}{2}}}{5^{\frac{1}{4}}} \\
 &= 5^{\frac{1}{2} + \frac{3}{4} - \frac{1}{4}} \times 2^{\frac{3}{2} + \frac{5}{2}} \\
 &= 5^{\frac{2}{4} + \frac{3}{4} - \frac{1}{4}} \times 2^{\frac{3}{2} + \frac{5}{2}} \\
 &= 5^{\frac{4}{4}} \times 2^{\frac{8}{2}} \\
 &= 5^1 \times 2^4 \\
 &= 5 \times 16 \\
 &= 80
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad & \frac{27^{y-2} \times 6^{y+1}}{162^y} \\
 &= \frac{(3^3)^{y-2} \times (2^1 \times 3^1)^{y+1}}{(2^1 \times 3^4)^y} \\
 &= \frac{3^{3y-6} \times 2^{y+1} \times 3^{y+1}}{2^y \times 3^{4y}} \\
 &= 3^{3y-6+y+1-4y} \times 2^{y+1-y} \\
 &= 3^{-5} \times 2^1
 \end{aligned}$$

$$= \frac{2}{3^5}$$

$$\begin{aligned}
 \text{(h)} \quad & \frac{3^{2p} + 3 \cdot 3^{2p}}{9^p - 2 \cdot 3^{2p+1}} \\
 &= \frac{3^{2p} + 3 \cdot 3^{2p}}{(3^2)^p - 2 \cdot 3^{2p+1}} \\
 &= \frac{1 \cdot 3^{2p} + 3 \cdot 3^{2p}}{1 \cdot 3^{2p} - 2 \cdot 3^{2p} \times 3^1} \\
 &= \frac{3^{2p}(1+3)}{3^{2p}(1-2 \times 3)} \\
 &= \frac{3^{2p}(1+3)}{3^{2p}(1-6)}
 \end{aligned}$$

$$= \frac{4}{-5}$$

(2) Solve for x :

(a) $25^x = 125$

$$\therefore 5^{2x} = 5^3$$

$$\text{EB} \Leftrightarrow \text{EE}$$

$$\therefore 2x = 3$$

$$\therefore x = \frac{3}{2}$$

(b) $2^{2x+1} = -4^x + 24$

$$\therefore 2^{2x} \times 2^1 = -2^{2x} + 24$$

$$\therefore 2^{2x} \times 2^1 + 1 \times 2^{2x} = 24$$

$$\therefore 2^{2x}(2 + 1) = 24$$

$$\therefore 2^{2x} \times 3 = 24$$

$$\therefore 2^{2x} = \frac{24}{3}$$

$$\therefore 2^{2x} = 8 = 2^3$$

$$\text{EB} \Leftrightarrow \text{EE}$$

$$\therefore 2x = 3$$

$$\therefore x = \frac{3}{2}$$

$$(c) \quad 3x^{\frac{3}{4}} = 24$$

$$\therefore x^{\frac{3}{4}} = \frac{24}{3} = 8$$

$$\therefore \left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = (8)^{\frac{4}{3}}$$

$$\therefore x^1 = 2^4$$

$$\therefore x = 16$$

$$(d) \quad (x^3 + 8)(2^{3x-1} - 1) = 0$$

$$\therefore x^3 + 8 = 0 \quad \text{or} \quad 2^{3x-1} - 1 = 0$$

$$\therefore x^3 = -8 \qquad 2^{3x-1} = 1$$

$$\therefore x^3 = (-2)^3 \qquad 2^{3x-1} = 2^0$$

$$EE \Leftrightarrow GB$$

$$EB \Leftrightarrow EE$$

$$\therefore x = -2$$

$$3x - 1 = 0$$

$$\therefore 3x = 1$$

$$\therefore x = \frac{1}{3}$$