Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

SEPTEMBER 2014

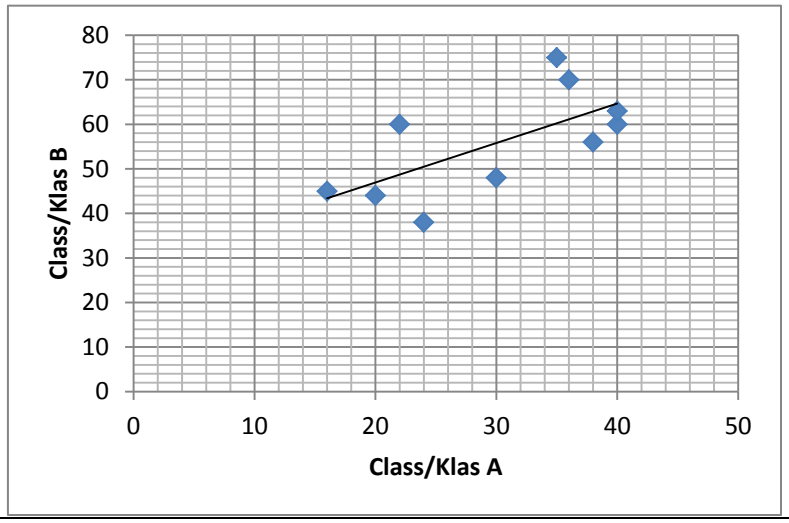
MATHEMATICS P2/*WISKUNDE V2* MEMORANDUM

MARKS/*PUNTE*: **150**

This memorandum consists of 12 pages./
Hierdie memorandum bestaan uit 12 bladsye

QUESTION/*VRAAG* 1			
1.1	16	✓answer/*antwoord*	(1)
1.2	Semi-IQR = $\frac{30-18,5}{2}$ = 5,75	✓ correct values *korrekte waardes* ✓ substitution into correct formula / *vervanging in korrekte formule* ✓ answer/*antwoord*	(3)
1.3	 	✓ Min and/*en* Max/*Maks* ✓ Q_1 & Q_3 ✓ Q_2	(3)
1.4	The distribution is skewed to the left/*Die verspreiding is skeef na links.*	✓ answer/*antwoord*	(1)
			[8]

QUESTION/*VRAAG* 2

2.1		✓✓ plotting points/ *afsteek van punte* ✓ regression line/ *regressielyn* (2.3)	(2)
2.2	$a = 29{,}22$ (29,21542…..) $b = 0{,}89$ $\therefore y = 29{,}22 + 0{,}89x$	✓ calculating value of *a* and *b* *bereken waardes van a en b* ✓ equation for the line/*vergelyking vir die lyn*	(2)
2.3	On graph/*Op grafiek*		(1)
2.4	$r = 0{,}66$ OR/*OF* $s_y = \sqrt{\frac{\sum(y-\bar{y})^2}{n}}$ $= \sqrt{\frac{1290{,}9}{10}} = 11{,}36$ $s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ $= \sqrt{\frac{720{,}9}{10}} = 8{,}49$ $b = r\frac{s_y}{s_x}$ then/*dan* $0{,}89 = r\frac{11{,}36}{8{,}49}$; $r = 0{,}66$	✓✓answer/*antwoord* ✓ S_y, S_x ✓ answer/*antwoord*	(2)
2.5	Mean/*Gemiddelde*, $\bar{x} = \frac{559}{10} = 55{,}90$ (Answer only: Full marks) *(Slegs antwoord: Volpunte)* Standard deviation/*Standaardafwyking*, $\sigma = 11{,}36$	✓✓answer/*antwoord* ✓✓answer/*antwoord*	(4)
2.6	55,9 – 11,36 and 55,9 + 11,36 [44,54; 67,26] 6 scores/*tellings*	✓interval/*interval* ✓answer/*antwoord*	(2)
			[13]

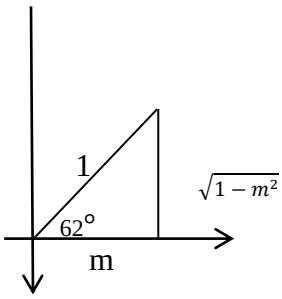
QUESTION/*VRAAG* 3			
3.1	$y = 9$	✓ answer/*antwoord*	(1)
3.2	AB = 11 units/*eenhede* **OR/*OF*** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(11 - 0)^2 + (9 - 9)^2}$ AB = 11 units/*eenhede*	✓ answer/*antwoord*	(1)
3.3	D(13; 9)	✓ answer/*antwoord*	(1)
3.4	Area of ΔABC = ½ AB.CD (*Oppervlakte van . . .*) = ½(11 × 8) = 44 square units/*vierkante eenhede*	✓ length of CD / *lengte van CD* ✓Substitute into correct formula / *Vervanging in korrekte formule* ✓answer/*antwoord*	(3)
3.5	$M = \left[\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right]$ $= \left[\frac{13}{2} ; \frac{10}{2}\right]$ = [6,5 ; 5]	✓substitution into midpoint formula/ *vervanging in middelpunt formule* ✓answer/*antwoord*	(2)
3.6	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9-1}{0-13}$ $m_{AC} = -\frac{8}{13}$ $y - y_1 = m(x - x_1)$ $y - 5 = \frac{13}{8}(x - 6{,}5)$ $y = \frac{13}{8}x - 5\frac{9}{16}$	✓ m_{AC} ✓ correct gradient of perpendicular bisector/*korrekte gradient van middelloodlyn* ✓ Substitution into correct formula/ *Vervanging in korrekte formule* ✓answer/*antwoord*	(4)
3.7	$y = \frac{13}{8}(11) - \frac{89}{16}$ $y = \frac{143}{8} - \frac{89}{16}$ $y = \frac{197}{16}$ No, it does not pass through B/*Nee, dit gaan nie deur B nie.*	✓substitution into correct equation / *vervanging in korrekte vergelyking* ✓ answer/*antwoord* (justification/ *regverdiging*)	(2)

3.8	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9-1}{11-13}$ $= \frac{8}{-2} = -4$ $\tan\theta = -4$ $\theta = 104{,}04°$ $A\hat{B}C = 104{,}04°$	✓ m_{BC} ✓ $\tan\theta$ ✓ answer/*antwoord*	(3)
3.9	$y - y_1 = m(x - x_1)$ $y - 9 = -\frac{8}{13}(x - 13)$ $y = -\frac{8}{13}x + 8 + 9$ $y = -\frac{8}{13}x + 17$	✓ substitution/*vervanging* ✓ answer/*antwoord*	(2)
			[19]

QUESTION/*VRAAG* 4			
4.1	$x^2 - 2x + y^2 + 4y = a$ $(x-1)^2 + (y+2)^2 = a + 1 + 4$ $(x-1)^2 + (y+2)^2 = a + 5$ $\therefore a + 5 = r^2$ $a + 5 = 25$ $a = 20$	✓✓completing the square/*voltooiing van vierkant* ✓equating r^2/*gelyk stel aan* r^2 ✓$r^2 = 25$	(4)
4.2	M (1; -2)	✓ answer in coordinate form/ *antwoord in koördinaat vorm*	(1)
4.3	$x^2 - 2x + y^2 + 4y = 20$ A(x; y) $\therefore (4)^2 - 2(4) + y^2 + 4y = 20$ $16 - 8 + y^2 + 4y = 20$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ $\therefore$ y = -6 or/of y = 2 $y = 2$ $A(4; 2)$ $y > 0$	✓ substituting into correct formula/*vervanging in korrekte formule* ✓standard form/*standaardvorm* ✓ factors/*faktore* ✓correct answer/*korrekte antwoord*	(4)
4.4	$m_{MA} = \frac{2+2}{4-1} = \frac{4}{3}$ $m_{tangent} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $\therefore y - 2 = -\frac{3}{4}(x - 4)$ $y = -\frac{3}{4}x + 3 + 2$ $\therefore y = -\frac{3}{4}x + 5$	✓$m_{MA} = \frac{4}{3}$ ✓$m_{tan.} = -\frac{3}{4}$ ✓substituting into correct formula/*vervanging in korrekte formule* ✓answer/*antwoord*	(4)

4.5	M(1; -2) , T(-1;-2) MT $=\sqrt{(-1-1)^2+(-2+2)^2}$ $= 2$ $2 < \sqrt{20}$ **OR/*OF*** $x^2 - 2x + y^2 + 4y = 20$ T (-1; -2) LHS $=(-1)^2 - 2(-1) + (-2)^2 + 4(-2)$ LHS = 1 + 2 + 4 – 8 LHS = -1 LHS ≠ RHS -1 < 20 ∴ T lies inside the circle/*T lê binne die sirkel*	✓ substitution/*vervanging* ✓simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
4.6	$(x-1)^2 + (y+2)^2 = 20$ $(x-1+3)^2 + (y+2-1)^2 = 20$ $(x+2)^2 + (y+1)^2 = 20$	✓ substitution into correct formula/*vervanging in korrekte formule* ✓ ✓ simplification and answer $(x + 2)$ and $(y + 1)$ / *vereenvoudiging en antwoord (x + 2) en (y + 1)*	(3)
			[19]

QUESTION/*VRAAG* 5

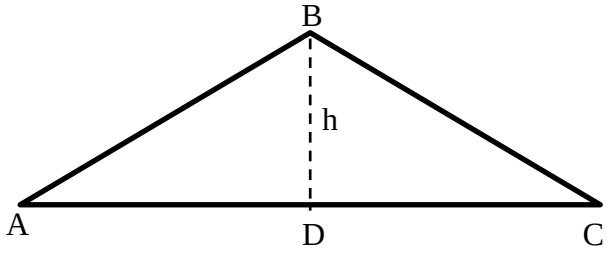
	5.1.1	$\sin 28° = \sin(90° - 62°)$ $= \cos 62°$ $= m$  	✓ equation/*vergelyking* ✓ answer/*antwoord*	(2)
	5.1.2	$\cos 362° = \cos(2°)$ $= \cos(62° - 60°)$ $= \cos 62° \cos 60° + \sin 62° \sin 60°$ $= \frac{1}{2}m + \frac{\sqrt{3}}{2}(\sqrt{(1-m^2)}$	✓ cos 2° ✓ 62° – 60° ✓ expansion/*uitbreiding* ✓substitution/*vervanging*	(4)
	5.2.1	$\frac{\tan(360° - x).\sin(90° + x)}{\sin(-x)}$ $= \frac{-\tan x.\cos x}{-\sin x}$ $= \frac{\frac{\sin x}{\cos x} \times \cos x}{\sin x}$ $= 1$	✓ - tan *x* ✓ cos *x* ✓ - sin *x* ✓ $\frac{sinx}{cosx}$ ✓ answer/*antwoord*	(5)

5.3	$4\sin^2\theta = 3$ $\sin^2\theta = \frac{3}{4}$ $\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$ $\therefore \theta = 120°;\ 90° < \theta < 180°$ $= \cos\frac{1}{4}\theta.\sin\frac{1}{2}\theta - \tan(3\theta - 45°)$ $= \cos 30°.\sin 60° - \tan(360° - 45°)$ $= \cos 30°.\sin 60° + \tan 45°$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 1$ $= 1\frac{3}{4}$	✓ simplification/*vereenvoudiging* ✓ value of/*waarde van* θ ✓substitution of/*vervanging van* θ ✓ Cos 30° = $\frac{\sqrt{3}}{2}$ ✓ Sin 60° = $\frac{\sqrt{3}}{2}$ ✓ tan 45° = 1	(6) **[17]**

QUESTION/*VRAAG* 6			
	LHS = $\frac{\cos 2x}{1+\sin 2x}$ = $\frac{\cos^2 x - \sin^2 x}{1+2\sin x\cos x}$ = $\frac{\cos^2 x - \sin^2 x}{\sin^2 x+2\sin x\cos x+\cos^2 x}$ = $\frac{(\cos x+\sin x)(\cos x-\sin x)}{(\sin x+\cos x)(\sin x+\cos x)}$ = $\frac{\cos x-\sin x}{\sin x+\cos x}$	✓ expansion in numerator/ *uitbreiding van teller* ✓ expansion in denominator/ *uitbreiding van noemer* ✓$\sin^2 x + \cos^2 x$ ✓factorisation/*faktorisering*	**[4]**

QUESTION/*VRAAG* 7

7.1



Let/*Laat* BD = h (height of triangle/*hoogte van driehoek* ABC)
Let/*Laat* DC = x
$\therefore$ AD = $(b - x)$
Proof/*Bewys*:
$c^2 = h^2 + (b - x)^2$
$= h^2 + b^2 - 2bx + x^2$
$= b^2 + (h^2 + x^2) - 2bx$
$= b^2 + a^2 - 2bx$
$a^2 = h^2 + x^2$
$\frac{x}{a} = cos\ C$
$x = a \cos C$
$\therefore\ b^2 + a^2 - 2ab \cos C$

✓AD in terms of x *AD in terme van x*
✓ c^2 (using/*gebruik van* Pythagoras)
✓ $(h^2 + x^2) = a^2$
✓substitution of x/ *vervanging van x*

(4)

7.2.1

PSQ = 90°
$Q_2 = 90° + \alpha$

In ΔQSR:
$SR^2 = SQ^2 + QR^2 - 2SQ.QR.\cos SQR$
$= x^2 + x^2 - 2x.x.\cos(90° + \alpha)$
$= 2x^2 - 2x^2(-\sin \alpha)$
$= 2x^2 + 2x^2(\sin \alpha)$
$= x^2(2 + 2\sin\alpha)$
$\therefore SR = x\sqrt{2(1 + \sin \alpha}$

✓ angle in semi-circle/*hoek in semi-sirkel*
✓ use of cos rule/ *gebruik van cos-reël*
✓ correct substitution/ *korrekte vervanging*
✓ $x^2 (2 + 2\sin\alpha)$

(4)

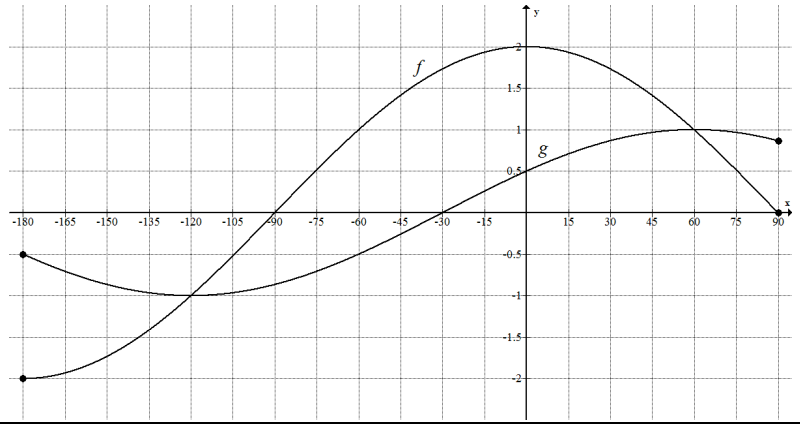
7.2.2

$5\sqrt{3} = x\sqrt{2(1 + sin\alpha)}$
$5\sqrt{3} = 5\sqrt{2(1 + \sin \alpha)}$
$\sqrt{3} = \sqrt{2(1 + \sin \alpha}$
$3 = 2(1 + sin\alpha)$
$\frac{3}{2} = 1 + sin\alpha$
$\frac{1}{2} = sin\alpha$
$\alpha = 30°$
In ΔPSQ
$\frac{QS}{PQ} = sin\ \alpha$
$PQ = \frac{QS}{\sin \alpha}$
$= \frac{5}{\sin 30°}$
= 10 units/*eenhede*

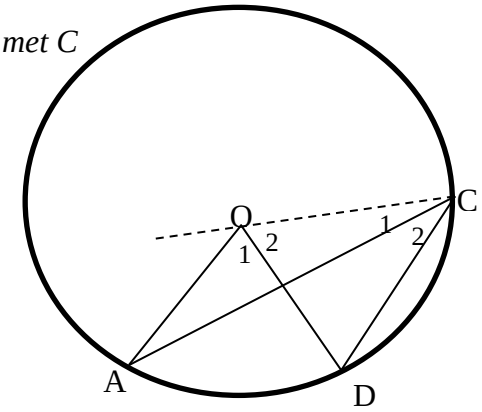
✓Substitution/ *Vervanging*
✓ $\sin\alpha = 0{,}5$
✓$\alpha = 30°$
✓ answer/*antwoord*

(4)

[12]

QUESTION/*VRAAG* 8			
8.1	2	✓ answer / *antwoord*	(1)
8.2		*f:* ✓ *x*- int/*x-afsnit* ✓ shape/*vorm* *g:* ✓ *x*-ints/*x-afsnitte* ✓*y*-int/*y-afsnit* ✓shape/*vorm*	(5)
8.3	$-120° \leq x \leq 60°$	✓ critical values / *kritiese waardes* ✓notation/*notasie*	(2)
8.4	Translate f 30° to the right/Skuif f 30^o na regs **OR/OF** Translate f down by ½ / *Skuif f 'n ½ eenheid af*	✓ correct translation/*korrekte translasie* ✓ correct translation/*korrekte translasie*	(2)
			[10]

QUESTION/*VRAAG* 9

9.1	Const: Join O to C *Konstr: Verbind O met C*  Construction/*Konstruksie*: Join CO produced/*Verbind CO verleng.* Proof/*Bewys*: Let/*Laat* $C_1 = x$ and/*en* $C_2 = y$ $\angle A = x$ [AO = OC= *r*] $\angle D = x + y$ [OD = OC= *r*] $\angle AOC = 180° - 2x$ [Sum of angles of/*Som v/d hoeke van* Δ] $\angle DOC = 180° - (2x + 2y)$ [Sum of angles of Δ / "] But/*Maar* $\angle AOD = \angle AOC - \angle DOC$ $\angle AOD = 2y$ $\angle AOD = 2\angle ACD$		✓ Construction/ *konstruksie* ✓ A and D (in terms of *x* and *y*) with reasons *A en D in terme van x en y met redes* ✓ AOC (in terms of *x*) with a reason *AOC in terme van x met rede* ✓ equation for AOD/ *gelykstel vir AOD* ✓ $\angle AOD = 2y$	(5)
9.2	9.2.1	MBC = MCB [MB = MC; radii] $\angle B_3 = C_2 = x$ [BO bisects/*halveer* MBC and/*en* OC bisects/*halveer* MCB] $\angle O_2 = 180^o - 2x$ [angles of/*hoeke van* Δ]	✓reason/*rede* ✓✓B_3 and/*en* $C_2 = x$ and reason/*en rede* ✓ answer/*antwoord*	(4)
	9.2.2	$\angle A = ½BMC$ [angle at centre = 2 × angle at circ /*middelpuntshoek = 2 × omtrekshoek*] $= ½(180° - 4x)$ [angles of/*hoeke van* Δ] $\angle A = 90° - 2x$ Therefore/*Daarom*: $\angle ABO = O_2 - A$ [ext. angle of/*buitehoek van* ΔOAB] $= (180° - 2x) - (90° - 2x)$ $= 90°$ AO subtends a right angle/*AO onderspan 'n reghoek* $\therefore$ AO is a diameter/*AO is 'n middellyn*	✓ $\angle A = ½BMC$ (statement) and reason. *(stelling) en rede* ✓ substitution and answer/*vervanging en antwoord* ✓ $\angle ABO = O_2 - A$ (statement) and reason *(stelling) en rede* ✓substitution/ *vervanging* $ABO = 90^o$ ✓ answer/*antwoord*	(5)

9.3	In ΔEHO, OE = x and OH = $x - 2$ $EH^2 = x^2 - (x-2)^2$ [Theorem of Pythagoras] $EH^2 = x^2 - x^2 + 4x - 4$ $= 4(x-1)$ $\therefore EH = 2\sqrt{x-1}$ $\therefore EF = 4\sqrt{x-1}$ line from centre of circle …/*lyn van middelpunt van sirkel* ...]	✓ OH = $x - 2$ ✓Theorem of/*Stelling van* Pythagoras ✓EH ✓EF [with reason/*met rede*]	(4)
			[19]

QUESTION/*VRAAG* 10			
10.1	∠ATB = 90° (subtended by diameter/*onderspan deur middellyn*) ∴ ∠ATB = ∠EPB = 90° ∴ TEPB is a cyclic quad/*is 'n koordevierhoek* (ext/*buite* ∠ = int opp/*teenoorst. binne* ∠)	✓∠ ATB = 90° ✓ ∠ ATB = ∠ EPB = 90° ✓conclusion with reason/ *gevolgtrekking met rede*	(3)
10.2	In ΔATB and/*en* ΔAPE ∠A = ∠A [common/*gemeen*] ∠ATB = ∠APE = 90° ∴ ΔATB /// ΔAPE (∠,∠,∠)	✓statement with reason / *stelling met rede* ✓ ∠ATB = ∠APE = 90° ✓ ΔATB /// ΔAPE with reason /*met rede*	(3)
10.3	∠RTA = ∠TBA (tangent chord/*raaklyn-koord*) ∠RTA = ∠ETP (vertically opposite/*regoorst*) ∠TBA = ∠TEP (ext/*buite* ∠ of cyclic quad/ *van koordevierhoek*) ΔTPE is Isosceles/*gelykbenig*. (∠ETP = ∠TEP) ∴ TP = PE	✓∠RTA = ∠TBA (statement) with reason. *(stelling) met rede* ✓∠RTA = ∠ETP (statement) with reason *(stelling) met rede* ✓statement with reason *(stelling) met rede* ✓ΔTPE is Isosceles/*gelykbenig* ✓ reason/*rede* ✓ conclusion/*slotsom*	(6)
10.4	∠ATB = ∠EPB = 90° ∠PTB = ∠A [tan chord/*raaklyn-koord*] ∠PTB = ∠PEB [subtended by BP/*onderspan deur BP*] ∴ ∠A = ∠ PEB Hence/*Vervolgens* ΔATB ///ΔEPB [equiangular/ *gelykhoekig*]	✓ ∠ATB =∠EPB = 90° ✓ ∠PTB = ∠A (statement) with reason *(stelling) met rede* ✓ ∠PTB = ∠PEB (statement) with reason *(stelling) met rede* ✓ ∠A = ∠PEB ✓ conclusion/*slotsom*	(5)
10.5	$\frac{BP}{TB} = \frac{BE}{AB}$ (ΔATB /// ΔEPB) AB.BP = BE.TB 2BP.BP = BE.TB [AO=OB=BP] $2BP^2$ = BE.TB	✓statement with reason *stelling met rede* ✓ AB.BP = BE.TB ✓2BP.BP = BE.TB (statement) with reason *(stelling) met rede* ✓answer/*antwoord*	(4)
			[21]

QUESTION/VRAAG 11				
11.1	Divides the two other sides proportionally/*verdeel die twee ander sye eweredig.*		✓ answer/*antwoord*	(1)
11.2	11.2.1	$\frac{KD}{DL} = \frac{2x}{x}$ $\frac{KE}{EM} = \frac{2y}{y}$ [ED // LM] LM ^{2} = KL^2 + KM ^{2} [Pythagoras] = $(3x)^2 + (3y)^2$ = $9x^2 + 9y^2$ = 9 DL^2 + 9 EM ^{2} …….1	✓✓ proportions / *eweredighede* ✓ Pythagoras ✓ answer/*antwoord*	(4)
	11.2.2	DM ^{2} + LE ^{2} = KM^2 + KD^2 + EK^2 + KL^2 = $(3y)^2 + (2x)^2 + (2y)^2 + (3x)^2$ = $9y^2 + 4x^2 + 4y^2 + 9x^2$ = $13y^2 + 13x^2$ = 13 EM^2 + 13 DL^2 = 13(EM^2 + DL^2) = $13 \times \frac{LM^2}{9}$ = $\frac{13}{9} LM^2$	✓ statement/*stelling* ✓substitution and simplification/ *vervanging en vereenvoudiging* ✓use of 1/*gebruik van 1* ✓ conclusion/*slotsom*	(4)
				[9]

TOTAL/*TOTAAL*: 150