Education and Sport Development

Department of Education and Sport Development
Departement van Onderwys en Sportontwikkeling
Lefapha la Thuto le Tlhabololo ya Metshameko

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GRADE 12

MATHEMATICS P1

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MEMORANDUM

MARKS: 150

This memorandum consists of 17 pages.

NOTE:

- If a candidate answered a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking memorandum.

QUESTION 1

1.1.1	$2x(5x-3)=0$ $2x=0$ or $5x-3=0$ $x=0$ $\quad 5x=3$ $x=\frac{3}{5}$	✓ $x=0$ ✓ $x=\frac{3}{5}$ (2)
1.1.2	$-x^2+4=5x$ $-x^2-5x+4=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{5\pm\sqrt{(-5)^2-4(-1)(4)}}{2(-1)}$ $=\frac{5\pm\sqrt{41}}{-2}$ $x=-5,70$ or $x=0,70$	✓ standard form ✓ substitution into the correct formula ✓ $x=-5,70$ ✓ $x=0,70$ (4)
1.1.3	$\sqrt{x-6}-2=\frac{15}{\sqrt{x-6}}$ *Let* $k=\sqrt{x-6}$ $k-2=\frac{15}{k}$ $k^2-2k-15=0$ $(k-5)(k+3)=0$ $k=5$ or $k=-3$ $\sqrt{x-6}=5$ or $\sqrt{x-6}=-3$ $x-6=25$ $\quad$ *n.a.* $x=31$	✓ $k^2-2k-15=0$ ✓ $(k-5)(k+3)=0$ ✓ $k=5$ or $k=-3$ ✓ $\sqrt{x-6}=-3$ *n.a.* ✓ $x=31$ (5)
1.1.4	$(x^2+2)(x-3)<0$ $(x^2+2)>0$ $\therefore (x-3)<0$ $x<3$	✓ $(x-3)<0$ ✓ $x<3$ (2)

1.2	$x + 2y = 3$ $x = 3 - 2y$ ....(1) $3x^2 + 4xy + 9y^2 - 16 = 0$ ....(2) $3(3-2y)^2 + 4(3-2y)y + 9y^2 - 16 = 0$ $3(9 - 12y + 4y^2) + 12y - 8y^2 + 9y^2 - 16 = 0$ $27 - 36y + 12y^2 + 12y - 8y^2 + 9y^2 - 16 = 0$ $13y^2 - 24y + 11 = 0$ $(13y - 11)(y - 1) = 0$ $y = \frac{11}{13}$ or $y = 1$ $x = 3 - 2\left(\frac{11}{13}\right)$ or $x = 3 - 2(1)$ $= \frac{17}{13}$ $= 1$	✓ $x = 3 - 2y$ ✓ substitution ✓ standard form ✓ factors or formula $y = \frac{24 \pm \sqrt{(-24)^2 - 4(13)(11)}}{2(13)}$ ✓ both y-values ✓ both x-values (6)
	OR $x + 2y = 3$ $2y = 3 - x$ $y = \frac{3 - x}{2}$ ....(1) $3x^2 + 4xy + 9y^2 - 16 = 0$ ....(2) $3x^2 + 4x\left(\frac{3-x}{2}\right) + 9\left(\frac{3-x}{2}\right)^2 - 16 = 0$ $3x^2 + 6x - 2x^2 + \frac{9(9 - 6x + x^2)}{4} - 16 = 0$ $12x^2 + 24x - 8x^2 + 81 - 54x + 9x^2 - 64 = 0$ $13x^2 - 30x + 17 = 0$ $(13x - 17)(x - 1) = 0$ $x = \frac{17}{13}$ or $x = 1$ $y = \frac{3 - \frac{17}{13}}{2}$ or $y = \frac{3-1}{2}$ $= \frac{11}{13}$ $= 1$	✓ $y = \frac{3-x}{2}$ ✓ substitution ✓ standard form ✓ factors or formula $x = \frac{30 \pm \sqrt{(-30)^2 - 4(13)(17)}}{2(13)}$ ✓ both x-values ✓ both y-values (6)

1.3	$2^{2022} . 5^{2018}$ $= 2^{2018+4} . 5^{2018}$ $= 2^{2018} . 2^4 . 5^{2018}$ $= (2.5)^{2018} . 16$ $= 10^{2018} . 16$ $= (100000000.....)(16)$ $= (160000000.....)$ Sum of the digits $= 1 + 6 = 7$	✓ $2^{2018} . 2^4 . 5^{2018}$ ✓ $(2.5)^{2018} . 16$ ✓ $(160000000.....)$ ✓ Sum of the digits = 7 (4) **[23]**

QUESTION 2

2.1.1	$\frac{T_2}{T_1} = \frac{T_3}{T_2}$ $\frac{x-1}{x+1} = \frac{2x-5}{x-1}$ $(x-1)^2 = (2x-5)(x+1)$ $x^2 - 2x + 1 = 2x^2 - 3x - 5$ $0 = x^2 - x - 6$ $0 = (x-3)(x+2)$ $x = 3$ or $x = -2$	✓ $\frac{x-1}{x+1} = \frac{2x-5}{x-1}$ ✓ simplification ✓ standard form ✓ factors ✓ both answers (5)
2.1.2	If $x = 3$: If $x = -2$: $R_1 : 4; 2; 1$ $R_2 : -1; -3; -9$ $r = \frac{1}{2}$ $r = 3$ For convergent: $-1 < r < 1$ $\therefore x = 3$	✓✓ both r-values ✓ $-1 < r < 1$ ✓ $x = 3$ (4)
2.1.3	$S_\infty = \frac{a}{1-r}$ $= \frac{4}{1-\frac{1}{2}}$ $= 8$	✓ substitution ✓ answer (2)

2.2	$1 + 2 + 4 + \ldots$ (30 terms) $S_n = \dfrac{a(r^n - 1)}{r - 1}$ $S_{30} = \dfrac{1(2^{30} - 1)}{2 - 1}$ $= 1\ 073\ 741\ 823$ $\therefore$ the 2nd option is the best	✓ substitution ✓ answer ✓ option 2 (3) **[14]**

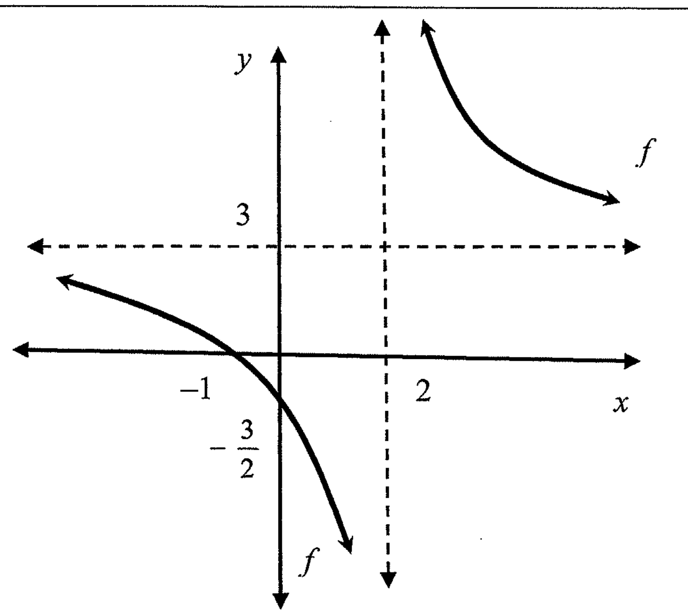
QUESTION 3

3.1	1 6 13 22 33 5 7 9 11 2 2 2 $2a = 2$ $a = 1$ $T_2 - T_1 = 3a + b$ $5 = 3(1) + b$ $b = 2$ $T_1 = a + b + c$ $1 = 1 + 2 + c$ $c = -2$ $T_n = n^2 + 2n - 2$	✓ first and second differences ✓ $a = 1$ ✓ $b = 2$ ✓ $c = -2$ (4)
3.2	$T_{15} = 15^2 + 2(15) - 2$ $= 253$ $\therefore 7^{th}$ chair in row $15 = 253 + 6$ $= 259$	✓ substitution ✓ answer (2)
3.3	Sequence of number of chairs: $5\ ;\ 7\ ;\ 9\ ;\ 11\ ;\ \ldots$ $T_n = a\ (n - 1)d$ $T_{25} = 5 + (25 - 1)(2)$ $= 53$ $\therefore$ 53 chairs in row 25 **OR**	✓ sequence ✓ substitution ✓ answer (3)

	Sequence of last chair of each row: $5 \quad 12 \quad 21 \quad 32 \quad 45$ $7 \quad 9 \quad 11 \quad 13$ $2 \quad 2 \quad 2$ $2a = 2$ $a = 1$ $T_2 - T_1 = 3a + b$ $7 = 3(1) + b$ $b = 4$ $T_1 = a + b + c$ $5 = 1 + 4 + c$ $c = 0$ $T_n = n^2 + 4n$ $T_{25} = (25)^2 + 4(25)$ $= 725$ $T_{24} = (24)^2 + 4(24)$ $= 672$ $\therefore$ Number of chairs in row $25 = 725 - 672$ $= 53$	✓ general term ✓ answer of both terms ✓ answer (3)
3.4	Sequence of number of chairs per row: 5; 7; 9; 11 $S_n = \frac{n}{2}[2a + (n-1)d]$ $2\,000 = \frac{n}{2}[2(5) + (n-1)(2)]$ $2\,000 = 5n + n^2 - n$ $0 = n^2 + 4n - 2\,000$ $n = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2\,000)}}{2(1)}$ $n = -46,77$ or $n = 42,77$ n.a. $\quad n = 42$ $\therefore$ 42 complete rows **OR**	✓ substitution ✓ standard form ✓ substitution into formula ✓ $n = 42,77$ ✓ $n = 42$ (5)

	Last chair of each row : 5; 7; 9; 11 $T_n = n^2 + 4n$ $2\,000 = n^2 + 4n$ $0 = n^2 + 4n - 2\,000$ $n = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-2\,000)}}{2(1)}$ $n = -46,77$ or $n = 42,77$ *n.a.* $n = 42$ $\therefore$ 42 complete rows	✓ substitution ✓ standard form ✓ substitution into formula ✓ $n = 42,77$ ✓ $n = 42$ (5) **[14]**

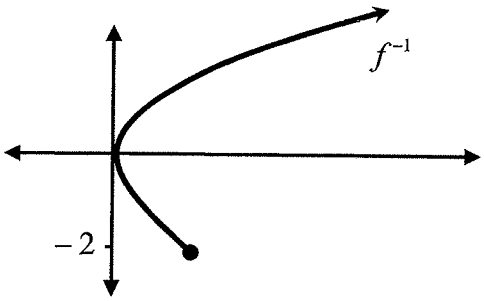
QUESTION 4

4.1	$f(x) = \dfrac{a}{x-2} + 3$ $0 = \dfrac{a}{-1-2} + 3$ $-3 = \dfrac{a}{-3}$ $9 = a$ $f(x) = \dfrac{9}{x-2} + 3$	✓ $f(x) = \dfrac{a}{x-2} + 3$ ✓ Substitution B(–1; 0) ✓ $a = 9$ (3)
4.2	$y = \dfrac{9}{0-2} + 3$ $= \dfrac{9}{-2} + 3$ $= -\dfrac{3}{2}$ $\therefore \left(0; -\dfrac{3}{2}\right)$ OR $y = -1\dfrac{1}{2}$ $\therefore \left(0; -1\dfrac{1}{2}\right)$	✓ $x = 0$ ✓ answer (2)
4.3		✓ x- and y-intercepts ✓ both asymptotes ✓ form (3) **[8]**

QUESTION 5

5.1	$x=-1$	✓ $x=-1$ (1)
5.2	$y=\frac{2}{0+1}-3$ $=2-3$ $=-1$ $\therefore P(0;-1)$	✓ $x=0$ ✓ $y=-1$ (2)
5.3	$f(x)=a(x+1)^2-3$ $-1=a(0-1)^2-3$ $a=2$ $f(x)=2(x+1)^2-3$	✓ $f(x)=a(x+1)^2-3$ ✓ $-1=a(0-1)^2-3$ ✓ $a=2$ (3)
5.4	$h(x)=-(x+1)-3$ $=-x-1-3$ $=-x-4$ **OR** $h(x)=-x+c$ $-3=-(-1)+c$ $c=-4$ $h(x)=-x-4$	✓ $h(x)=-(x+1)-3$ ✓ answer (2) ✓ $-3=-(-1)+c$ ✓ answer (2)
5.5	$h(x)=-x-4$ $k>3$	✓✓ $k>3$ (2)
5.6	$m(x)=\frac{2}{2x+1}-3+5$ $=\frac{2}{2\left(x+\frac{1}{2}\right)}+2$ $=\frac{1}{x+\frac{1}{2}}+2$ $x\in R; x\neq-\frac{1}{2}$	✓ $m(x)=\frac{2}{2x+1}-3+5$ ✓ $\frac{1}{x+\frac{1}{2}}+2$ ✓ $x\in R; x\neq-\frac{1}{2}$ (3) **[13]**

QUESTION 6

6.1	$g(x) = a^x$ $3 = a^{-1}$ $3 = \frac{1}{a}$ $a = \frac{1}{3}$	✓ $3 = a^{-1}$ (1)
6.2	$[-2; 0)$ OR $-2 \le x < 0$	✓✓ $[-2; 0)$ OR ✓✓ $-2 \le x < 0$ (2)
6.3	$f: \quad y = \frac{1}{4}x^2; \; x \ge -2; \; y \ge 0$ $f^{-1}: x = \frac{1}{4}y^2$ $4x = y^2$ $y = \pm\sqrt{4x} = \pm 2\sqrt{x}\,; \; x \ge 0$	✓ swop x and y ✓ $y = \pm\sqrt{4x} = \pm 2\sqrt{x}$ (2)
6.4		✓ form ✓ y-value of end point (2)
6.5	$y \ge -2$	✓ notation ✓ -2 (2)
6.6	$g: \quad y = \left(\frac{1}{3}\right)^x$ $g^{-1}: x = \left(\frac{1}{3}\right)^y$ $y = \log_{\frac{1}{3}} x; \; x > 0$	✓ swop x and y ✓ $y = \log_{\frac{1}{3}} x$ (2)
6.7	$x \in (0; 3]$	✓ (0; ✓ 3] (2) **[13]**

QUESTION 7

7.1	$P=\dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $2\ 000\ 000=\dfrac{35\ 000\left[1-\left(1+\frac{0,093}{12}\right)^{-n}\right]}{\frac{0,093}{12}}$ $\dfrac{31}{70}=1-\left(1+\dfrac{0,093}{12}\right)^{-n}$ $\left(1+\dfrac{0,093}{12}\right)^{-n}=\dfrac{39}{70}$ OR $\log\left(1+\dfrac{0,093}{12}\right)^{-n}=\log\dfrac{39}{70}$ $-n=\log_{\left(1+\frac{0,093}{12}\right)}\dfrac{39}{70}$ $\quad -n=\dfrac{\log\frac{39}{70}}{\log\left(1+\frac{0,093}{12}\right)}$ $n=75,77$ $\therefore$ He will be able to live for 75 months from his investment	✓ substitution of i ✓ substitution into correct formula ✓ simplification ✓ correct use of log ✓ answer (5)
7.2.1	$A=P(1+i)^n$ $=1\ 200\ 000(1+0,075)^6$ $=R1\ 851\ 961,83$ He will need $=R\ 1\ 851\ 961,83-400\ 000$ $=R\ 1\ 451\ 961,83$	✓ substitution ✓ $R1\ 851\ 961,83$ ✓ answer (3)
7.2.2	$A=P(1+i)^n$ $1\ 451\ 961,83=P\left(1+\dfrac{0,11}{4}\right)^2$ $P=1\ 375\ 281,30$ $F=\dfrac{x\left[(1+i)^n-1\right]}{i}$ $1\ 375\ 281,30=\dfrac{x\left[\left(1+\frac{0,11}{4}\right)^{21}-1\right]}{\frac{0,11}{4}}$ $x=R49\ 261,76$ **OR**	✓ substitution ✓ substitution of i ✓ substitution of n ✓ substitution into correct formula ✓ answer (5)

	$F=\frac{x(1+i)^n\left[(1+i)^n-1\right]}{i}$ $1\ 451\ 961,83=\frac{x\left(1+\frac{0,11}{4}\right)^2.\left[\left(1+\frac{0,11}{4}\right)^{21}-1\right]}{\frac{0,11}{4}}$ $x=R49\ 261,76$	✓ $\left(1+\frac{0,11}{4}\right)^2$ ✓ substitution of i ✓ substitution of n ✓ substitution into correct formula ✓ answer (5)
7.2.3	Quarterly deposit to withdraw R 8 000 at the end of each year from year 2 to year 5: $F=\frac{x\left[(1+i)^n-1\right]}{i}$ $8\ 000=\frac{x\left[\left(1+\frac{0,11}{4}\right)^4-1\right]}{\frac{0,11}{4}}$ $x=R1\ 919,36$ New quarterly deposit $=49\ 261,76+1\ 919,36$ $=R\ 51\ 181,12$	✓ substitution of n ✓ substitution into correct formula ✓ $x=R1\ 919,36$ ✓ answer (4) **[17]**

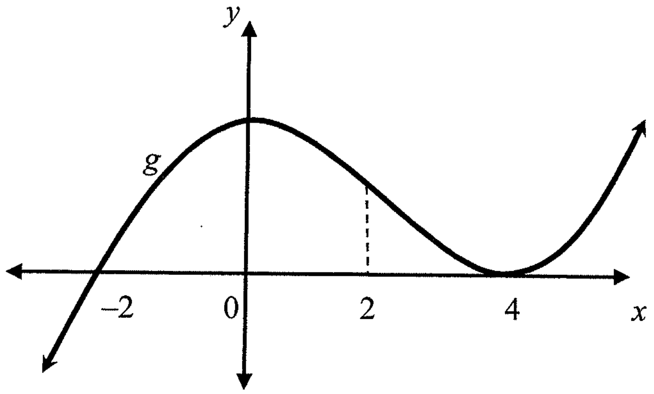
QUESTION 8

8.1	$f(x)=2x^2-5x+3$ $f(x+h)=2(x+h)^2-5(x+h)+3$ $=2(x^2+2xh+h^2)-5x-5h+3$ $=2x^2+4xh+2h^2-5x-5h+3$ $f(x+h)-f(x)=2x^2+4xh+2h^2-5x-5h+3-(2x^2-5x+3)$ $=4xh+2h^2-5h$ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to0}\frac{4xh+2h^2-5h}{h}$ $=\lim_{h\to0}\frac{h(4x+2h-5)}{h}$ $=\lim_{h\to0}(4x+2h-5)$ $=4x-5$	✓ $2(x+h)^2-5(x+h)+3$ ✓ $4xh+2h^2-5h$ ✓ formula ✓ factor ✓ answer (5)

	OR $f(x) = 2x^2 - 5x + 3$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) + 3 - (2x^2 - 5x + 3)}{h}$ $= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 5x - 5h + 3 - 2x^2 + 5x - 3}{h}$ $= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h + 3 - 2x^2 + 5x - 3}{h}$ $= \lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h}$ $= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$ $= \lim_{h \to 0} (4x + 2h - 5)$ $= 4x - 5$	✓ formula ✓ $2(x+h)^2 - 5(x+h) + 3$ ✓ $4xh + 2h^2 - 5h$ ✓ factor ✓ answer (5)
8.2	$y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3}$ $= \frac{2x^2}{3x^{\frac{1}{2}}} - 2 - \frac{1}{x^3}$ $= \frac{2}{3}x^{\frac{3}{2}} - 2 - x^{-3}$ $\frac{dy}{dx} = x^{\frac{1}{2}} + 3x^{-4}$	✓ $\frac{2}{3}x^{\frac{3}{2}}$ ✓ -2 ✓ $-x^{-3}$ ✓ $x^{\frac{1}{2}}$ ✓ $3x^{-4}$ (5) **[10]**

QUESTION 9

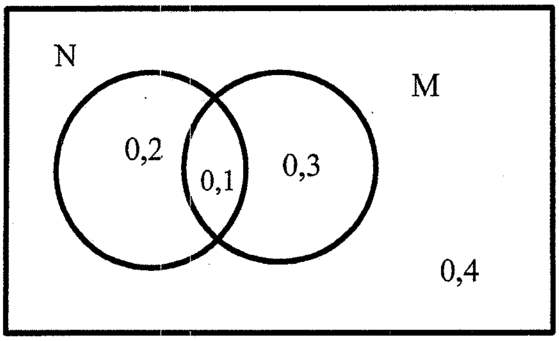
9.1.1	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $0 = (x-3)(-2x^2 - x + 1)$ $x - 3 = 0$ or $-2x^2 - x + 1 = 0$ $x = 3$ $\quad 2x^2 + x - 1 = 0$ $(3;0)$ $\quad (2x-1)(x+1) = 0$ $2x = 1$ or $x = -1$ $x = \frac{1}{2}$ $\quad (-1;0)$ $\left(\frac{1}{2};0\right)$	✓ $-2x^2 - x + 1$ ✓ $(3;0)$ ✓ factors or formula ✓ both coordinates $\left(\frac{1}{2};0\right)$ $(-1;0)$ (4)

9.1.2	$f'(x) = -6x^2 + 10x + 4$ $0 = -6x^2 + 10x + 4$ $3x^2 - 5x - 2 = 0$ $(3x + 1)(x - 2) = 0$ or $x = \dfrac{-10 \pm \sqrt{10^2 - 4(-6)(4)}}{2(-6)}$ $3x = -1$ or $x = 2$ $x = -\dfrac{1}{3}$	✓ $-6x^2 + 10x + 4$ ✓ $f'(x) = 0$ ✓ factors or formula ✓ both x answers (4)
9.1.3	$f''(x) = -12x + 10$ $0 = -12x + 10$ $12x = 10$ $x = \dfrac{10}{12} = \dfrac{5}{6}$ $\therefore x < \dfrac{5}{6}$	✓ $0 = -12x + 10$ ✓ $x < \dfrac{5}{6}$ (2)
9.2.1		✓ form ✓ both x-intercepts ✓ x-value of both turning points ✓ x-value of point of inflection (4)
9.2.2	$x < -2$ or $x > 2$; $x \neq 4$	✓ $x < -2$ ✓ $x > 2$ ✓ $x \neq 4$ (3) **[17]**

QUESTION 10

10.1	$N(t) = t^3 - 12t^2 + 36t + 8$ $N(0) = 8$ $\therefore$ 8 people	✓ answer (1)
10.2	$N'(t) = 3t^2 - 24t + 36$ increasing $N'(t) \geq 0$ $3t^2 - 24t + 36 \geq 0$ $t^2 - 8t + 12 \geq 0$ $(t-6)(t-2) \geq 0$ $t \geq 6$ or $t \leq 2$ $\therefore$ for first 2 hours after opening or 6 hours after opening until closing time	✓ $N'(t) = 3t^2 - 24t + 36$ ✓ $N'(t) \geq 0$ ✓ $(t-6)(t-2)$ ✓ $t \geq 6$ ✓ $t \leq 2$ (5)
10.3	Minimum turning point at $t = 6$ hours after opening	✓ $t = 6$ (1) **[7]**

QUESTION 11

11.1.1	 **OR** $P(M \text{ or } N) = P(M) + P(N) - P(M \text{ and } N)$ $0,6 = 0,4 + 0,3 - P(M \text{ and } N)$ $P(M \text{ and } N) = 0,4 + 0,3 - 0,6$ $= 0,1$	✓ $P(N \text{ and } M') = 0,2$ ✓ $P(M \text{ and } N') = 0,3$ ✓ $P(N \text{ or } M') = 0,4$ ✓✓ $P(M \text{ and } N) = 0,1$ (5) ✓ $P(N \text{ and } M') = 0,2$ ✓ $P(M \text{ and } N') = 0,3$ ✓ $P(N \text{ or } M') = 0,4$ ✓ substitution into formula ✓ $P(M \text{ and } N) = 0,1$ (5)
11.1.2	$LH = P(M \text{ and } N)$ $= 0,1$ $RH = P(M) \times P(N)$ $= (0,4)(0,3)$ $= 0,12$ $\therefore P(M \text{ and } N) \neq P(M) \times P(N)$ $\therefore$ M and N is not independent	✓ P(M and N) = 0,1 ✓ P(M) . P(N) = (0,4)(0,3) ✓ 0,12 ✓ $P(M \text{ and } N) \neq P(M) \times P(N)$ ✓ No, not independent (5)

11.2	$1 \times 8 \times 7 \times 6 \times 2$ $= 672$ **OR** $(1 \times 8 \times 7 \times 6 \times 1) + (1 \times 8 \times 7 \times 6 \times 1)$ $= 672$	✓ $1 \times$ ✓ $8 \times 7 \times 6$ ✓ $\times 2$ ✓ 672 (4) ✓ $1 \times$ ✓ $8 \times 7 \times 6 \times 1$ ✓ $+ (1 \times 8 \times 7 \times 6 \times 1)$ ✓ 672 (4) **[14]**
		TOTAL: 150