education

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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

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MARKING GUIDELINES

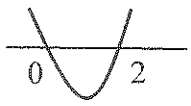
MARKS: 150

These marking guidelines consist of 16 pages and two pages with cognitive levels.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in ALL aspects of the marking memorandum.

QUESTION 1

1.1.1	$9x^2 - 7x - 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(9)(-3)}}{2(9)}$ $= \frac{7 \pm \sqrt{157}}{18}$ $x = 1{,}08$ or $x = -0{,}31$	✓ substitution into the correct formula ✓ $x = 1{,}08$ ✓ $x = -0{,}31$ (3)
1.1.2	$5x^2 - 10x > 0$ $5x(x - 2) > 0$ $5x < 0$ or $x > 2$ $\therefore x < 0$ or $x > 2$ 	✓ factors/critical values ✓✓ $x < 0$ or $x > 2$ (3)
1.1.3	$4 - \sqrt{x + 5} = x + 3$ $4 - x - 3 = \sqrt{x + 5}$ $(1 - x)^2 = x + 5$ $1 - 2x + x^2 = x + 5$ $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4$ or $x = -1$ *n.a.*	✓ root the subject ✓ square both sides ✓ standard form ✓ factors/formula ✓ both answers ✓ selection (6)
1.2.1	$y = 4: (x - 3)(4 + 4) = 0$ $(x - 3) = 0$ $x = 3$	✓ $x = 3$ (1)
1.2.2	$y = -4: (x - 3)(-4 + 4) = 0$ $(x - 3)(0) = 0$ $x \in \mathbb{R}$	✓ $x \in \mathbb{R}$ (1)

1.3	$2y + x = 1$ $x = 1 - 2y$ $x^2 + y^2 = y - x$ $(1 - 2y)^2 + y^2 = y - (1 - 2y)$ $1 - 4y + 4y^2 + y^2 = y - 1 + 2y$ $5y^2 - 7y + 2 = 0$ $(5y - 2)(y - 1) = 0$ $5y = 2$ or $y = 1$ $y = \frac{2}{5}$ $x = 1 - 2\left(\frac{2}{5}\right)$ $x = 1 - 2(1)$ $= \frac{1}{5}$ $= -1$ **OR** $2y = 1 - x$ $y = \frac{1 - x}{2}$ $x^2 + \left(\frac{1 - x}{2}\right)^2 = \frac{1 - x}{2} - x$ $x^2 + \left(\frac{1 - 2x + x^2}{4}\right) = \frac{1 - x}{2} - x$ $4x^2 + 1 - 2x + x^2 = 2(1 - x) - 4x$ $4x^2 + 1 - 2x + x^2 = 2 - 2x - 4x$ $5x^2 + 4x - 1 = 0$ $(5x - 1)(x + 1) = 0$ $5x = 1$ or $x = -1$ $x = \frac{1}{5}$ $y = \frac{2}{5}$ $y = 1$	✓ $x = 1 - 2y$ ✓ substitution ✓ standard form ✓ factors/formula ✓ both y-values ✓ both x-values (6) **OR** ✓ $y = \frac{1 - x}{2}$ ✓ substitution ✓ standard form ✓ factors/formula ✓ both x-values ✓ both y-values (6)
1.4	$5x^2 - kx + 16 = (x + 2).Q(x) + 10$ Say $f(x) = 5x^2 - kx + 16$ $f(-2) = 5(-2)^2 - k(-2) + 16$ $10 = 20 + 2k + 16$ $-26 = 2k$ $-13 = k$	✓ substitution ✓ $f(-2) = 10$ ✓ answer (3) **[23]**

QUESTION 2

2.1.1	1; 1,25; 1,5; ... $T_n = a + (n-1)d$ $T_{10} = 1 + (9)(0,25)$ $= 3,25$ hours	✓ sequence ✓ substitution ✓ answer (3)
2.1.2	$6 = 1 + (n-1)(0,25)$ $5 = (n-1)(0,25)$ $20 = n - 1$ $21 = n$ $\therefore$ on 21st day	✓ substitution ✓ answer (2)
2.1.3	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{21} = \frac{21}{2}\left[2(1) + (21-1)(0,25)\right]$ $= 73,5$ Day 22 to 30 $= (9)(6)$ $= 54$ Total hours $= 73,5 + 54$ $= 127,5$ *hours*	✓ substitution ✓ 73,5 ✓ 54 ✓ answer (4)
2.2	AS: $\log x + \log y + \log z$ $\therefore T_2 - T_1 = T_3 - T_2$ $\log y - \log x = \log z - \log y$ $\log \frac{y}{x} = \log \frac{z}{y}$ $\frac{y}{x} = \frac{z}{y}$ $\therefore \frac{T_2}{T_1} = \frac{T_3}{T_2}$ $\therefore$ Geometric sequence: $x; y; z$	✓ $\log y - \log x = \log z - \log y$ ✓ $\log \frac{y}{x} = \log \frac{z}{y}$ ✓ $\frac{y}{x} = \frac{z}{y}$ ✓ $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ (4) **[13]**

QUESTION 3

3.1.1	$64 + 32 + 16 + \ldots$ $r = \frac{32}{64} = \frac{1}{2}$ $T_n = ar^{n-1}$ $T_9 = 64\left(\frac{1}{2}\right)^{9-1}$ $= \frac{1}{4}$	✓ $r = \frac{1}{2}$ ✓ substitution ✓ answer (3)
3.1.2	$S_\infty = \frac{a}{1-r}$ $= \frac{64}{1-\frac{1}{2}}$ $= 128$	✓ substitution ✓ answer (2)
3.2.1	1st differences : $T_n = 6n + 8$ 14 20 26 1st differences 6 6 2nd difference $2a = 6$ $a = 3$	✓ 1st differences ✓ 2nd difference ✓ $2a = 6$ (3)
3.2.2	$T_2 - T_1 = 3a + b$ $14 = 3(3) + b$ $5 = b$ $T_1 = a + b + c$ $2 = 3 + 5 + c$ $-6 = c$ $\therefore T_n = 3n^2 + 5n - 6$	✓ $b = 5$ ✓ $c = -6$ ✓ $T_n = 3n^2 + 5n - 6$ (3)
3.3	$\sum_{r=5}^{17} (3r + 2)\, p^{r+3}\, k^{20-r}$ **OR** $\sum_{r=8}^{20} (3r - 7)\, p^{r}\, k^{23-r}$ **OR** $\sum_{r=1}^{13} (3r + 14)\, p^{r+7}\, k^{16-r}$	✓ $\sum_{r=begin}^{end}$ ✓ general term ✓ exponent of p ✓ exponent of k (4) **[15]**

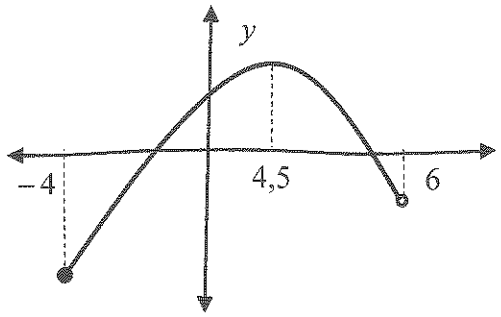
QUESTION 4

4.1	$f(x) = -x^2 - 6x - 4$ $x = -\frac{b}{2a}$ OR $f'(x) = -2x - 6$ $= -\frac{(-6)}{2(-1)}$ $0 = -2x - 6$ $= -3$ $x = -3$ $y = -(-3)^2 - 6(-3) - 4$ $= 5$ $\therefore A(-3; 5)$	✓ substitution / $f'(x) = 0$ ✓ $x = -3$ ✓ $y = 5$ (3)
4.2	$B(0; -4)$	✓ $y = -4$ (1)
4.3	$f(x) = -x^2 - 6x - 4$ $0 = -x^2 - 6x - 4$ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(-1)(-4)}}{2(-1)}$ $x = -0,76$ or $x = -5,24$	✓ $y = 0$ ✓ substitution ✓ both x answers (3)
4.4	$g(x) = \frac{2}{x+3} - 4$	✓ $x + 3$ ✓ $\frac{2}{x+3} - 4$ (2)
4.5	$y = (x + 3) - 4$ $= x + 3 - 4$ $= x - 1$ **OR** $y = x + c$ $-4 = -3 + c$ $c = -1$ $y = x - 1$	✓ substitution ✓ answer (2) **OR** ✓ substitution ✓ answer (2)

4.6	$x-1=\frac{2}{x+3}-4$ $x+3=\frac{2}{x+3}$ $(x+3)^2=2$ $x^2+6x+9=2$ $x^2+6x+7=0$ $x=\frac{-6\pm\sqrt{6^2-4(1)(7)}}{2(1)}$ $x=-1,59$ or $x=-4,41$ *n.a.* $y=-1,59-1$ $=-2,59$ $\therefore(-1,59;-2,59)$	✓ equating ✓ simplify ✓ standard form ✓ $x=-1,59$ ✓ y answer (5)
4.7	$y-y_1=m(x-x_1)$ $y+2,59=-(x+1,59)$ $y=-x-1,59-2,59$ $y=-x-4,18$	✓ $m=-1$ ✓ substitution ✓ answer (3)
4.8	$[-5,24;-3]$ or $[-0,76;\infty)$ **OR** $-5,24\le x\le -3$ or $-0,76\le x$	✓ $[-5,24;-3]$ ✓ $[-0,76;\infty)$ **OR** ✓ $-5,24\le x\le -3$ ✓ $-0,76\le x$ (2) **[21]**

QUESTION 5

5.1	$0 = -\frac{2}{3}x + 3$ $\frac{2}{3} = 3$ $x = \frac{9}{2} = 4,5$	✓ $y = 0$ ✓ $x = \frac{9}{2} = 4,5$ (2)
5.2	$k(x) = -\frac{2}{3}x + 3;\ -4 \leq x < 6$ $k(-4) = -\frac{2}{3}(-4) + 3$ $k(6) = -\frac{2}{3}(6) + 3$ $= 5,67$ $= -1$ $\therefore -1 < x \leq 5,67$ **OR** $x \in (-1; 5,67]$	✓ critical values ✓ notation (2) **OR** ✓ critical values ✓ notation (2)
5.3	$h:\ y = \left(\frac{1}{2}\right)^x$ $h^{-1}: x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$ **OR** $h:\ y = 2^{-x}$ $h^{-1}: x = 2^{-y}$ $-y = \log_2 x$ $y = -\log_2 x$	✓ swop x and y ✓ answer **OR** ✓ swop x and y ✓ answer (2)
5.4	$(1; 0)$	✓ $x = 1$ ✓ $y = 0$ (2)
5.5	$3 < x \leq 5,67$ **OR** $x \in (3; 5,67]$	✓ critical values ✓ notation (2) **OR** ✓ critical values ✓ notation (2)

5.6		✓ turning point at $x = 4,5$ ✓ x-values of end points ✓ form (3) **[13]**

QUESTION 6

6.1	$F = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ $3\ 000\ 000 = \dfrac{x\left[\left(1+\dfrac{0,1}{12}\right)^{241} - 1\right]}{\dfrac{0,1}{12}}$ $x = R\ 3\ 912,89$	✓ $n = 241$ ✓ $i = \dfrac{0,1}{12}$ ✓ substitution in correct formula ✓ answer (4)

6.2	$A = P(1+i)^n$ $= 3\ 000\ 000\left(1+\frac{0,08}{12}\right)^3$ $= R3\ 060\ 400,889$	✓ substitution in correct formula ✓ answer
	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $3\ 060\ 400,889 = \frac{20\ 600\left[1-\left(1+\frac{0,08}{12}\right)^{-n}\right]}{\frac{0,08}{12}}$	✓ substitution in correct formula
	$0,9904209997 = 1-\left(1+\frac{0,08}{12}\right)^{-n}$ $\left(1+\frac{0,08}{12}\right)^{-n} = 1-0,9904209997$	✓ simplification
	$\log_{\left(1+\frac{0,08}{12}\right)}(0,009579) = -n$ OR $\frac{\log(0,009579)}{\log\left(1+\frac{0,08}{12}\right)} = -n$	✓ correct use of logs
	$n = 699,548824061$ ∴ He will survive 702 months after his retirement on his current lifestyle.	✓ $n = 699,55$ ✓ answer (7)

6.3	$A = P(1+i)^n$ $= 3\,000\,000\left(1+\frac{0,08}{12}\right)^{702}$ $= 318\,327\,675,10$ OR $= 3\,060\,400,889\left(1+\frac{0,08}{12}\right)^{699}$ $= 318\,327\,675,10$ $F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $= \frac{20\,600\left[\left(1+\frac{0,08}{12}\right)^{699} - 1\right]}{\frac{0,08}{12}}$ $= 318\,316\,427,40$ Outstanding amount after month 699 $= A - F$ $= 318\,327\,675,10 - 318\,316\,427,40$ $= R11\,247,73$ Last withdrawal: $A = P(1+i)^n$ $= 11\,247,73\left(1+\frac{0,08}{12}\right)^{1}$ $= R11\,322,72$ **OR** $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{20\,600\left[1-\left(1+\frac{0,08}{12}\right)^{-0,548824061}\right]}{\frac{0,08}{12}}$ $= R11\,247,73546$ Last withdrawal: $A = P(1+i)^n$ $= 11\,247,73\left(1+\frac{0,08}{12}\right)^{1}$ $= R11\,322,72$	✓ substitution in correct formula ✓ substitution in correct formula ✓ A - F ✓ answer (4) **OR** ✓ n = – 0,548824061 ✓ substitution in correct formula ✓ answer ✓ answer (4) **[15]**

QUESTION 7

7.1	$f(x) = -x^2 + 7x + 9$	
	$f(x+h) = -(x+h)^2 + 7(x+h) + 9$	✓ $-(x+h)^2 + 7(x+h) + 9$
	$= -(x^2 + 2xh + h^2) + 7x + 7h + 9$	
	$= -x^2 - 2xh - h^2 + 7x + 7h + 9$	
	$f(x+h) - f(x) = (-x^2 - 2xh - h^2 + 7x + 7h + 9)$	
	$- (-x^2 + 7x + 9)$	
	$= -2xh - h^2 + 7h$	✓ simplification
	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
	$= \lim_{h \to 0} \frac{-2xh - h^2 + 7h}{h}$	✓ substitution in formula
	$= \lim_{h \to 0} \frac{h(-2x - h + 7)}{h}$	✓ factors
	$= \lim_{h \to 0} (-2x - h + 7)$	
	$= -2x + 7$	✓ answer (5)
	OR	**OR**
	$f(x) = -x^2 + 7x + 9$	
	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
	$= \lim_{h \to 0} \frac{-(x+h)^2 + 7(x+h) + 9 - (-x^2 + 7x + 9)}{h}$	✓ $-(x+h)^2 + 7(x+h) + 9$ ✓ substitution in formula
	$= \lim_{h \to 0} \frac{-(x^2 + 2xh + h^2) + 7x + 7h + 9 + x^2 - 7x - 9}{h}$	
	$= \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 + 7x + 7h + 9 + x^2 - 7x - 9}{h}$	
	$= \lim_{h \to 0} \frac{-2xh - h^2 + 7h}{h}$	✓ simplification
	$= \lim_{h \to 0} \frac{h(-2x - h + 7)}{h}$	✓ factors
	$= \lim_{h \to 0} (-2x - h + 7)$	
	$= -2x + 7$	✓ answer (5)
7.2	$f(x) = \frac{4}{x^2} + 3x^5$	
	$= 4x^{-2} + 3x^5$	✓ $4x^{-2}$
	$f'(x) = -8x^{-3} + 15x^4$	✓ $-8x^{-3}$ ✓ $+15x^4$ (3)

7.3	$\frac{y}{x-3} = 1 + x$ $y = (x-3)(1+x)$ $= x + x^2 - 3 - 3x$ $= x^2 - 2x - 3$ $\frac{dy}{dx} = 2x - 2$	✓ y subject ✓ simplification ✓ answer (3) **[11]**

QUESTION 8

8.1	$y = a(x - x_1)(x - x_2)(x - x_3)$ $= a(x-3)^2(x+2)$ $9 = a(0-3)^2(0+2)$ $9 = a(9)(2)$ $a = \frac{1}{2}$ $g(x) = \frac{1}{2}(x-3)^2(x+2)$ $= \frac{1}{2}(x^2 - 6x + 9)(x+2)$ $= \frac{1}{2}(x^3 - 6x^2 + 9x + 2x^2 - 12x + 18)$ $= \frac{1}{2}(x^3 - 4x^2 - 3x + 18)$ $= \frac{1}{2}x^3 - 2x^2 - \frac{3}{2}x + 9$ $a = \frac{1}{2}\ \ b = -2\ \ c = -\frac{3}{2}\ d = 9$	✓ substitution roots ✓ substitution (0; 9) ✓ $a = \frac{1}{2}$ ✓ simplification (4)
8.2	$g'(x) = \frac{3}{2}x^2 - 4x - \frac{3}{2}$ $0 = \frac{3}{2}x^2 - 4x - \frac{3}{2}$ $0 = 3x^2 - 8x - 3$ $x = \frac{8 \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$ or $0 = (x-3)(3x+1)$ $x = 3$ or $x = -\frac{1}{3}$ *n.a.*	✓ $g'(x) = \frac{3}{2}x^2 - 4x - \frac{3}{2}$ ✓ $g'(x) = 0$ ✓ formula/factors ✓ $x = -\frac{1}{3}$ (4)

8.3	$g''(x) = 3x - 4$ $0 = 3x - 4$ $4 = 3x$ $\frac{4}{3} = x$ $\therefore$ concave up : $x > \frac{4}{3}$	✓ $g''(x) = 3x - 4$ ✓ $g''(x) = 0$ ✓ answer (3)
8.4	$\frac{3}{2}x^2 - 4x - \frac{3}{2} = -\frac{7}{2}$ $3x^2 - 8x - 3 = -7$ $3x^2 - 8x + 4 = 0$ $(x - 2)(3x - 2) = 0$ $x = 2$ or $x = \frac{2}{3}$ $y = 2$ *n.a.*	✓ $g'(x) = -\frac{7}{2}$ ✓ standard form ✓ factors/formula ✓ $x = 2$ ✓ $y = 2$ (5) **[16]**

QUESTION 9

9.1	$4r + 2h = 16$ $2r + h = 8$ $h = 8 - 2r$	✓ equating ✓ simplification (2)
9.2	$V = \pi r^2 h$ $= \pi r^2(8 - 2r)$ $= 8\pi r^2 - 2\pi r^3$	✓ formula ✓ substitution ✓ answer (3)
9.3	$V'(r) = 16\pi r - 6\pi r^2$ $0 = 16\pi r - 6\pi r^2$ $0 = 8\pi r - 3\pi r^2$ $0 = r(8\pi - 3\pi r)$ $r = 0$ or $8\pi - 3\pi r = 0$ *n.a.* $8\pi = 3\pi r$ $r = \frac{8}{3}$ $h = 8 - 2\left(\frac{8}{3}\right)$ $= \frac{8}{3}$	✓ $V'(r) = 16\pi r - 6\pi r^2$ ✓ $V'(r) = 0$ ✓ factors ✓ $r = \frac{8}{3}$ ✓ $h = \frac{8}{3}$ (5) **[10]**

QUESTION 10

10.1	$P(\text{Tom and Jerry}) = 0,85 \times 0,67$ $= 0,5695$ $\approx 0,57$	✓ $0,85 \times 0,67$ ✓ answer (2)
10.2	$P(\text{Tom and not Jerry}) = 0,85 \times 0,33$ $= 0,2805$ $\approx 0,28$	✓ $0,85 \times 0,33$ ✓ answer (2)
10.3	$P(\text{At least one})$ $= 1 - P(\text{none})$ $= 1 - P(\text{not Tom and not Jerry})$ $= 1 - (0,15)(0,33)$ $= 1 - 0,0495$ $= 0,9505$ $\approx 0,95$ **OR** $P(\text{At least one})$ $= P(\text{Tom and not Jerry}) \text{ or } P(\text{Jerry and not Tom})$ $\text{or } P(\text{Tom and Jerry})$ $= (0,85 \times 0,33) + (0,67 \times 0,15) + (0,85 \times 0,67)$ $= 0,2805 + 0,1005 + 0,5695$ $= 0,9505$ $\approx 0,95$	✓ $1 - P(\text{none})$ ✓ $P(\text{none})$ ✓ answer (3) **OR** ✓ $(0,85 \times 0,33) + (0,67 \times 0,15)$ ✓ $(0,85 \times 0,67)$ ✓ answer (3) [7]

QUESTION 11

11.1	Number of 5 – digit numbers in the bottle $= (1 \times 10 \times 10 \times 10 \times 10) - 1$ $= 10\,000 - 1$ $= 9\,999$	✓ 10 000 ✓ 9 999 (2)
11.2	Number of 5 – digit numbers that does not repeat: $= 1 \times 9 \times 8 \times 7 \times 6$ $= 3\,024$ Number of 5 – digit numbers that does not repeat without a 6: $= 1 \times 8 \times 7 \times 6 \times 5$ $= 1\,680$ Number of 5 – digit numbers with at least one 6 : $= 3\,024 - 1\,680$ $= 1\,344$ P(5 – digit number with at least one 6) $= \frac{1\,344}{9\,999}$ $= 0{,}13$	✓ 3 024 ✓ 1 680 ✓ 1 344 ✓ 0,13 (4) [6]
		TOTAL: 150

COGNITIVE LEVELS **MATHEMATICS P1**

	COGNITIVE LEVELS				TOPICS						
	LEVEL 1 (20%)	LEVEL 2 (35%)	LEVEL 3 (30%)	LEVEL 4 (15%)							
QUESTION	KNOWLEDGE	ROUTINE PROCEDURES	COMPLEX PROCEDURES	PROBLEM SOLVING	ALGEBRA	PATTERNS	FUNCTIONS	FINANCE	DIFFERENTIATION	PROBABILITY	TOTAL MARKS
1.1.1	3				3						
1.1.2		3			3						
1.1.3		6			6						
1.2.1	1				1						
1.2.2	1				1						
1.3		6			6						
1.4			3		3						23
2.1.1	3					3					
2.1.2		2				2					
2.1.3			4			4					
2.2				4		4					
3.1.1	3					3					13
3.1.2	2					2					
3.2.1			3			3					
3.2.2			3			3					
3.3				4		4					15
4.1	3						3				
4.2	1						1				
4.3	3						3				
4.4	2						2				
4.5	2						2				
4.6		5					5				
4.7		3					3				
4.8			2				2				21
5.1		2					2				
5.2		2					2				
5.3		2					2				
5.4	2						2				
5.5				2			2				
5.6			3						3		13
6.1		4						4			
6.2			7					7			
6.3				4				4			15
7.1		5							5		
7.2	3								3		
7.3			3						3		11
8.1		4							4		
8.2		4							4		
8.3		3							3		
8.4			5						5		16

	COGNITIVE LEVELS				TOPICS						
	LEVEL 1 (25%)	LEVEL 2 (30 %)	LEVEL 3 (30%)	LEVEL 4 (15%)							
QUESTION	KNOWLEDGE	ROUTINE PROCEDURES	COMPLEX PROCEDURES	PROBLEM SOLVING	ALGEBRA	PATTERNS	FUNCTIONS	FINANCE	DIFFERENTIATION	PROBABILITY	TOTAL MARKS
9.1			2						2		
9.2				3					3		
9.3				5					5		10
10.1	2									2	
10.2		2								2	
10.3			3							3	7
11.1			2							2	
11.2			4							4	6
TOT	**31**	**53**	**44**	**22**	**23**	**28**	**34**	**15**	**37**	**13**	**150**
%	**21%**	**35%**	**29%**	**15%**							
Pol	**20%**	**35%**	**30%**	**15%**	**25**	**25**	**35**	**15**	**35**	**15**	**150**