



PROVINSIALE EKSAMEN
NOVEMBER 2022
GRAAD 11
NASIENRIGLYNE

WISKUNDE (VRAESTEL 2)

22 bladsye

INSTRUKSIES EN INLIGTING

- **A** – AKKURAATHEID
- **CA** – KONTINUE AKKURAATHEID
- **S** – BEWERING
- **R** – REDE
- **S/R** – BEWERING met REDE

NOTAS:

- Indien 'n kandidaat 'n vraag TWEE keer antwoord, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en nie die vraag weer gedoen het nie, merk die doodgetrekte poging.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyn toegepas word.
- Dit is ONAANVAARBAAR vir kandidate om waardes/antwoorde aan te neem in 'n poging om probleme op te los.

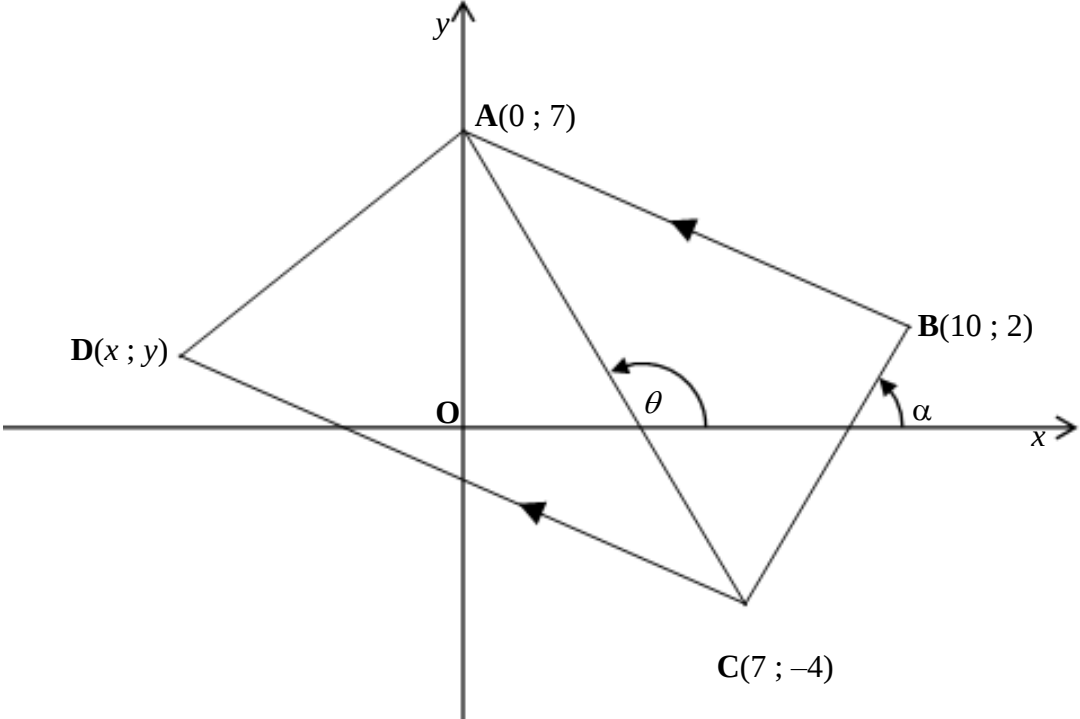
VRAAG 1

1.1		✓ Mond (Q1 en Q3) ✓ Snor (Min. en Maks.) ✓ Mediaan	(3)
1.2	Die data is simmetries versprei, 'n normale verspreiding. OF Die data is effens skeef na regs.	✓ Verduideliking	(1)
1.3	$\sigma = 23,77$	✓✓ σ	(2)
1.4	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $= (287 - 23,77; 287 + 23,77)$ $= (263,23; 310,77)$ $\therefore$ 6 munisipaliteite is binne een standaard afwyking.	✓ $\bar{x} = 287$ ✓ Interval ✓ 6	(3)
1.5	$S-IQR = \frac{1}{2} (Q_3 - Q_1)$ $= \frac{1}{2} (306 - 266)$ $= 20$	✓ $Q_3 - Q_1$ ✓ $\frac{1}{2} \times$ ✓ Antwoord	(3)
			[12]

VRAAG 2

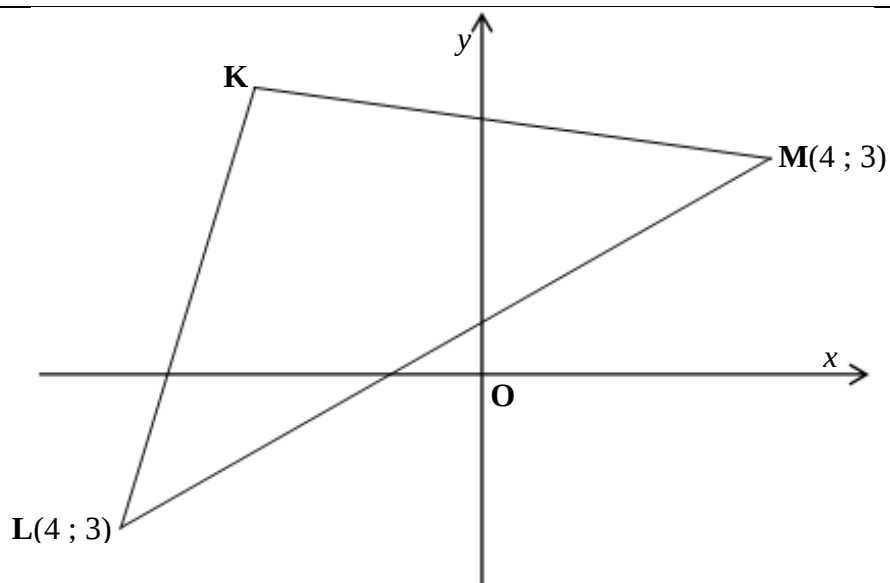
2.1	$p = 5$ $q = 27$ $r = 1$	✓ p ✓ q ✓ r	(3)
2.2	KUMULATIEWE FREKWENSIEGRAFIEK (OGIEF) ✓ Grond by (20 ; 0) ✓ Eindpunte ✓ Verbind met vryhand		(3)
2.3	$M = 46$	✓✓ $M = 46(\pm 2)$	(2)
2.4	$32 - 24$ $= 8$ $\therefore 8$ leerders behaal 'n punt van meer as 55.	✓ $KF = 24(\pm 1)$ ✓ $32 - KF$	(2)
			[10]

VRAAG 3

			
3.1	$AB = \sqrt{(0-10)^2 + (7-2)^2}$ $AB = \sqrt{125}$ $AB = 5\sqrt{5}$	✓ Vervang in korrekte formule ✓ AB ✓ Eenvoudigste wortelvorm	(3)
3.2	$m_{AC} = \frac{7-(-4)}{0-7}$ $m_{AC} = -\frac{11}{7}$	✓ Gradiënt substitusie ✓ m_{AC}	(2)
3.3	$\tan \theta = -\frac{11}{7}$ $\text{verw.hoek} = 57,528 \dots^\circ$ $\theta = 180^\circ - 57,528 \dots^\circ$ $\theta = 122,47^\circ$	✓ Inklinasie hoek vervang (CA 4.2) ✓ Verw.hoek ✓ θ (trek verw.hoek af van 180°)	(3)

3.4	$\tan \alpha = 2$ $\alpha = 63,434 \dots^\circ$ $\hat{A}CB = 122,47^\circ - 63,434 \dots^\circ$ $\hat{A}CB = 59,04^\circ$ $m_{AB} = \frac{1}{2}$ $\hat{C}AB = 180^\circ - \tan^{-1} \left(\frac{1}{2} \right) - 122,74^\circ$ $\hat{C}AB = 30,69^\circ$ $\hat{A}CD = 30,69^\circ$ [verw. $\angle$ 'e, $AB \parallel CD$] $\hat{BCD} = 30,69^\circ + 59,04^\circ$ $\hat{BCD} = 89,73^\circ$ OF Indien θ nie afgerond is nie $\tan \alpha = 2$ $\alpha = 63,434 \dots^\circ$ $\hat{A}CB = 122,4711 \dots^\circ - 63,434 \dots^\circ$ $\hat{A}CB = 59,036 \dots^\circ$ $m_{AB} = \frac{1}{2}$ $\hat{C}AB = 180^\circ - \tan^{-1} \left(\frac{1}{2} \right) - 122,74 \dots^\circ$ $\hat{C}AB = 30,963^\circ$ $\hat{A}CD = 30,963^\circ$ [verw $\angle$ 'e, $AB \parallel CD$] $\hat{BCD} = 30,963^\circ + 59,036 \dots^\circ$ $\hat{BCD} = 90^\circ$	α $\hat{A}CB$ $\hat{C}AB$ $\hat{BCD}$ OF $\checkmark \alpha$ $\checkmark \hat{A}CB$ $\checkmark \hat{C}AB$ $\checkmark \hat{BCD}$	(4)
			[12]

VRAAG 4



4.1	$y = 5x + 9$ en $5y + x - 19 = 0$ $5(5x + 9) + x = 19$ $26x = -26$ $x = -1$ $y = 5(-1) + 9$ $y = 5$ $\therefore K(-1; 4)$	✓ Vervang y ✓ Los op vir x ✓ Vervang x terug GEEN PUNTE vir koördinate nie (gegee)	(3)
4.2	$m_{KL} = 5$ en $m_{KM} = -\frac{1}{5}$ $\therefore m_{KL} \times m_{KM} = -1$ $KL \perp KM$ $\therefore \hat{LKM} = 90^\circ$	✓ m_{KL} ✓ m_{KM} ✓ Definisie van loodregte gradiënte	(3)

4.3	$KL = \sqrt{((-1 - (-2))^2 + (4 - (-1))^2)}$ $KL = \sqrt{26}$ $KN = \sqrt{(-1 - 4)^2 + (4 - 3)^2}$ $KN = \sqrt{26}$ $\text{Area } \triangle KML = \frac{1}{2} (\sqrt{26})(\sqrt{26})$ $\text{Area } \triangle KML = 13 \text{ eenhede}^2$	✓ KL lengte ✓ KN lengte ✓ Vervang in Oppervlak formule ✓ Opp $\triangle KML$	(4)
4.4	$M \left(\frac{-2 + 4}{2}; \frac{3 + (-1)}{2} \right)$ $M(1; 1)$ $m_{NL} = \frac{3 - (-1)}{4 - (-2)}$ $m_{NL} = \frac{2}{3}$ $y - y_1 = -\frac{2}{3}(x - x_1)$ $y - 1 = -\frac{2}{3}(x - 1)$ $y = -\frac{2}{3}x + \frac{5}{2}$	✓ Middelpunt van NL ✓ Gradiënt van NL ✓ Loodregte gradiënt ✓ Vervang Middelpunt ✓ Vergelyking	(5)
4.5	$m_{NL} = m_{NP} = m_{LP}$ $\frac{2}{3} = \frac{y - 3}{7 - 4}$ $y = 5$	✓ Definisie van kollineêr ✓ Vervang $P(7; y)$	(2)

4.6	$Q(3; -2)$	$\checkmark x_Q$ $\checkmark y_Q$	(2)
4.7	$\hat{K} = 90^\circ$ [bewys in 4.1] $KL = KN = \sqrt{26}$ [bewys in 4.2] $\therefore KLQN$ is 'n vierkant [’n parallelogram met ’n binnehoek van 90° en aangrensende sye gelyk]	$\checkmark$ Binnehoek van 90° $\checkmark$ Aangrensende sye gelyk	(2)
			[21]

VRAAG 5

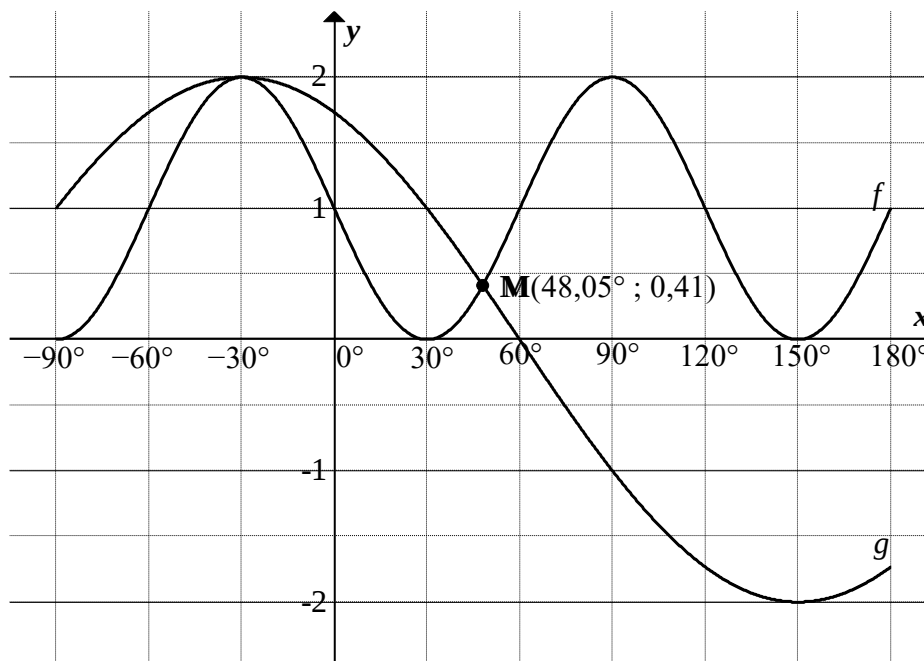
5.1	$\frac{\tan(180^\circ - x) \cos(360^\circ + x)}{2 \cos(90^\circ + x)}$ $= \frac{-\tan x \cdot \cos x}{-2 \sin x}$ $= \frac{-\frac{\sin x}{\cos x} \cdot \cos x}{-2 \sin x}$ $= \frac{-\sin x}{-2 \sin x}$ $= \frac{1}{2}$		$\checkmark -\tan x$ $\checkmark \cos x$ $\checkmark -2 \sin x$ $\checkmark -\tan x$ identiteit $\checkmark$ vereenvoudig $\checkmark$ antwoord	(6)
5.2	5.2.1	$LK = \frac{\sin \theta - 2 \sin \theta \cos \theta}{2 \cos^2 \theta + \cos \theta - 1}$ $= \frac{\sin \theta (1 - 2 \cos \theta)}{(2 \cos \theta - 1)(\cos \theta + 1)}$ $= \frac{-\sin \theta (2 \cos \theta - 1)}{(2 \cos \theta - 1)(\cos \theta + 1)}$ $= \frac{-\sin \theta}{\cos \theta + 1}$ $= RK$	$\checkmark \sin \theta (1 - 2 \cos \theta)$ $\checkmark (2 \cos \theta - 1)(\cos \theta + 1)$ $\checkmark$ Teken verander/negatiewe faktor $\checkmark$ Vereenvoudig <u>met</u> gevolgtrekking	(4)

5.2.2	Ongedefinieerd wanneer $2\cos\theta - 1 = 0$ $\cos\theta = \frac{1}{2}$ Verw. hoek = 60° QI : $\theta = 60^\circ + k.360^\circ ; k \in \mathbb{Z}$ QIV : $\theta = 300^\circ + k.360^\circ$ OF wanneer $\cos\theta = -1$ $\theta = 180^\circ + k.360^\circ ; k \in \mathbb{Z}$ NOTA: Aanvaar ander ekwivalente algemene oplossings $\cos\theta - 1 = 0$ $\theta = 180^\circ + k.360^\circ ; k \in \mathbb{Z}$	✓ $2\cos\theta - 1 = 0$ ✓ QI algemene oplossing ✓ QIV algemene oplossing ✓ $k \in \mathbb{Z}$ ✓ $\cos\theta = -1$ algemene oplossing	(5)
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5.3				
5.3.1	$b^2 + 1^2 = 4^2$ $b = \pm \sqrt{15}$ $b = -\sqrt{15} \quad (\text{QII})$		✓ Pythagoras substitusie ✓ $b = -\sqrt{15}$	(2)
5.3.2	(a)	$\tan \alpha = -\frac{1}{\sqrt{15}}$	✓ $\frac{y}{x}$ ✓ CA x	(2)
	(b)	$\sin(-\alpha - 180^\circ) + \cos(-\alpha)$ $= \sin \alpha + \cos \alpha$ $= \left(\frac{1}{4}\right) + \left(\frac{-\sqrt{15}}{4}\right)$ $= \left(\frac{1 - \sqrt{15}}{4}\right)$	✓ $\sin \alpha$ ✓ $\cos \alpha$ ✓ Vervang $x ; y$ en r	(3)

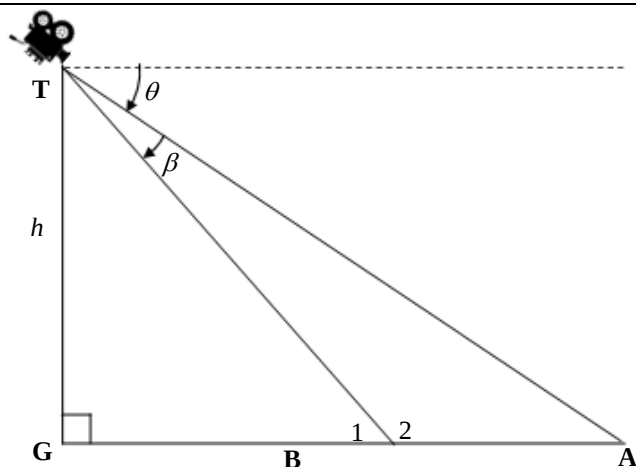
5.3.3	$\cos(\alpha - 90) = \frac{k}{6}$ $\sin \alpha = \frac{k}{6}$ $\therefore \frac{k}{6} = \frac{1}{4}$ $\therefore k = \frac{3}{2}$	$\sin(\alpha - 90) = \frac{m}{6}$ $-\cos \alpha = \frac{m}{6}$ $\cos \alpha = -\frac{m}{6}$ $\therefore \frac{m}{6} = \left(\frac{-\sqrt{15}}{4} \right)$ $\therefore m = \frac{3\sqrt{15}}{2}$	$\checkmark \cos(\alpha - 90) = \frac{k}{6}$ $\checkmark \sin(\alpha - 90) = \frac{m}{6}$ $\checkmark$ Komplementêre herleiding $\checkmark$ Verhoudings gelyk stel $\checkmark$ Waardes van k en m	
	OF P lê in QI, daarom is k en m positief 'n 90° rotasie beteken $(x ; y) \rightarrow (y ; x)$ $\frac{r_2}{r_1}$ $\frac{6}{4}$ $\frac{3}{2}$ $\therefore P\left(\frac{3}{2}; \frac{3}{2}\sqrt{15}\right) = P(k ; m)$ $\therefore k = \frac{3}{2} \text{ en } m = \frac{3\sqrt{15}}{2}$	OF $\checkmark$ P in QI $\checkmark (x ; y) \rightarrow (y ; x)$ idee $\checkmark$ Verhouding van radii $\checkmark$ Pas skaal toe op x en y $\checkmark$ Waardes van k en m Antwoord alleen MAG NIE volpunte kry NIE.		
				(5)
				[27]

VRAAG 6



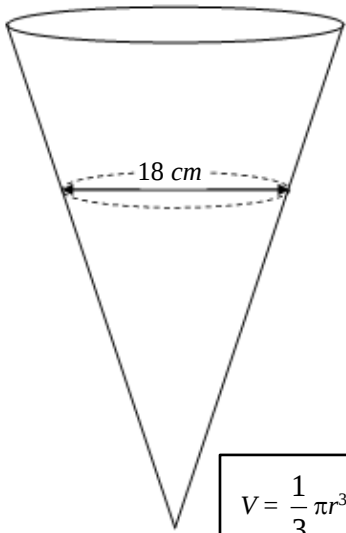
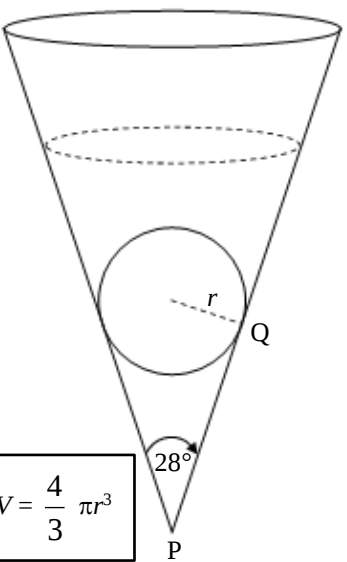
6.1	A = 2		✓ Antwoord	(1)
6.2	120°		✓ Antwoord	(1)
6.3	a = −1 b = 3 p = 30°		✓ a ✓ b ✓ p	(3)
6.4	6.4.1	−90° ≤ x < 60°	✓ Interval	(1)
	6.4.2	x = −30° of 48,05° ≤ x ≤ 180°	✓ Interval ✓ x = −30°	(2)
	6.4.3	60° < x < 150° of 150° < x ≤ 180°	✓ 60° en 180° ✓ Nie ingesl. 150° maar 180° ingesl.	(2)
6.5	t = −30° + k. 360° ; k ∈		✓✓ t = −30° ✓ k. 360°	(3)
				[13]

VRAAG 7



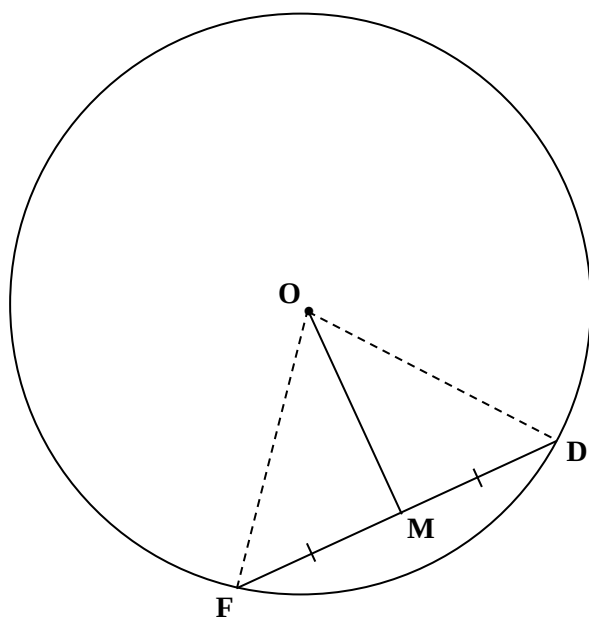
7.1	$\hat{B}_1 = \theta = \beta$	✓ Antwoord	(1)
7.2	In $\triangle TBG$ $\sin(\theta + \beta) = \frac{h}{BT}$ $BT = \frac{h}{\sin(\theta + \beta)}$ In $\triangle ABT$ $\hat{A} = \theta$ $\frac{x}{\sin \beta} = \frac{BT}{\sin \theta}$ $\frac{x}{\sin \beta} = \frac{h}{\sin(\theta + \beta) \sin \theta}$ $x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$	✓ Korrekte verhouding in $\triangle TBG$ ✓ BT ✓ Sin reël substitusie ✓ BT substitusie ✓ vereenvoudiging	(5)
7.3	$x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$ $x = \frac{20 \sin 37^\circ}{\sin(65^\circ + 37^\circ) \sin 65^\circ}$ $x = 13,58 \text{ m}$ Die kat stap 13,58 m	✓ Substitusie ✓ Antwoord	(2)
			[8]

VRAAG 8

FIGUUR A		FIGUUR B	
			
8.1	$V = \frac{1}{3} \pi (9)^3$ $V = 763,41 \text{ cm}^3$	✓ Substitusie ($r = 9$) ✓ Antwoord	(2)
8.2	$V = \frac{4}{3} \pi r^3$ $268,08 = \frac{4}{3} \pi r^3$ $\sqrt[3]{\frac{3 \times 268,08}{4\pi}} = r$ $r = 4$ $\tan 14^\circ = \frac{4}{PQ}$ $PQ = \frac{4}{\tan 14^\circ}$ $PQ = 16,04 \text{ cm}$	✓ Vervang volume in die korrekte formule ✓ Onderwerp van die formule ✓ r ✓ $\tan 14^\circ$ ✓ PQ	(5)
			[7]

VRAAG 9

9.1



✓ Konstruksie

*Konstruksie: Verbind OF en OD*In $\triangle OFM$ en $\triangle ODM$ i. $FM = MD$ [GEGEE]ii. OM is 'n gemeenskaplike syiii. $OF = OD$ [Radii] $\therefore \triangle OFM \equiv \triangle ODM$ [SSS]

✓ R

 $\hat{M}_1 = \hat{M}_2$ [$\triangle OFM \equiv \triangle ODM$]

✓ R

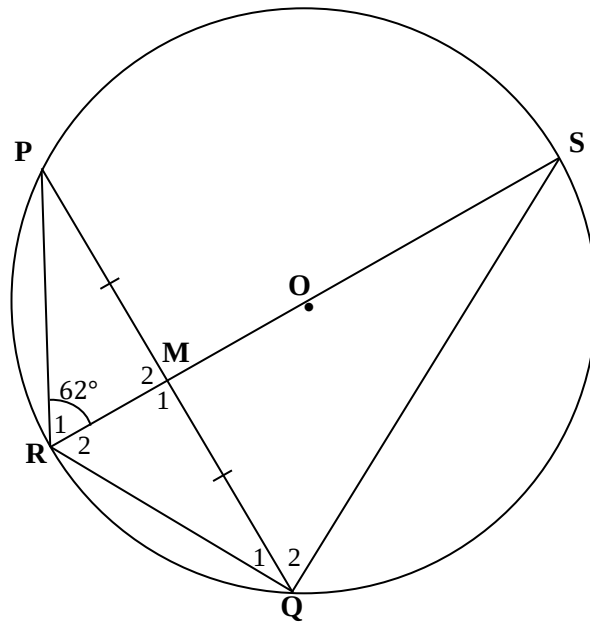
Maar $\hat{M}_1 + \hat{M}_2 = 180^\circ$ [$\angle$ 'e op 'n reguit lyn]

✓ S

 $\therefore \hat{M}_1 = \hat{M}_2 = 90^\circ$ $\therefore OM \perp FD$

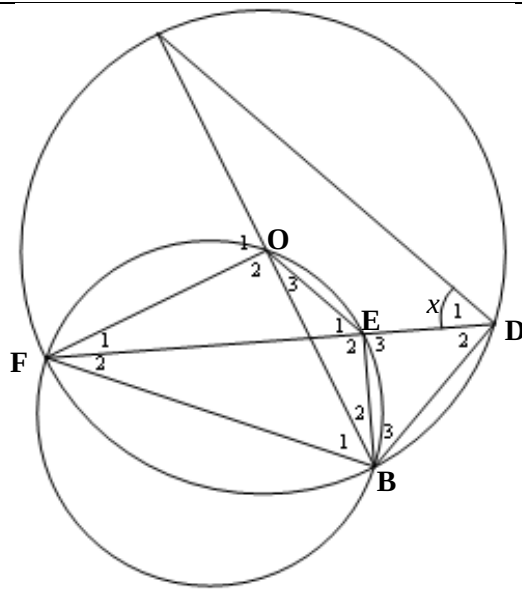
(4)

9.2

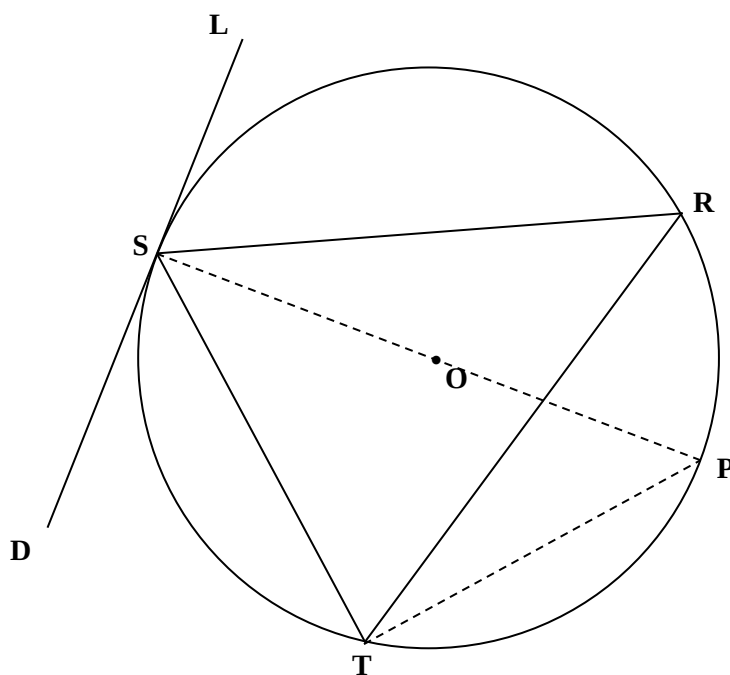


9.2.1	$\hat{M}_2 = 90^\circ$	[lyn van midpunt sirkel, tot midpunt koord]	✓ S ✓ R	(2)
9.2.2	$\hat{Q}_1 + \hat{Q}_2 = 90^\circ$	[$\angle$ 'e in 'n halwe sirkel]	✓ S ✓ R	(2)
9.2.3	$\hat{P} = 28^\circ$	[binne $\angle$ 'e van 'n Δ]	✓ S/R	
	$\hat{S} = \hat{P} = 28^\circ$	[$\angle$ 'e in dieselfde sirkelsegment]	✓ R	(2)
				[10]

VRAAG 10

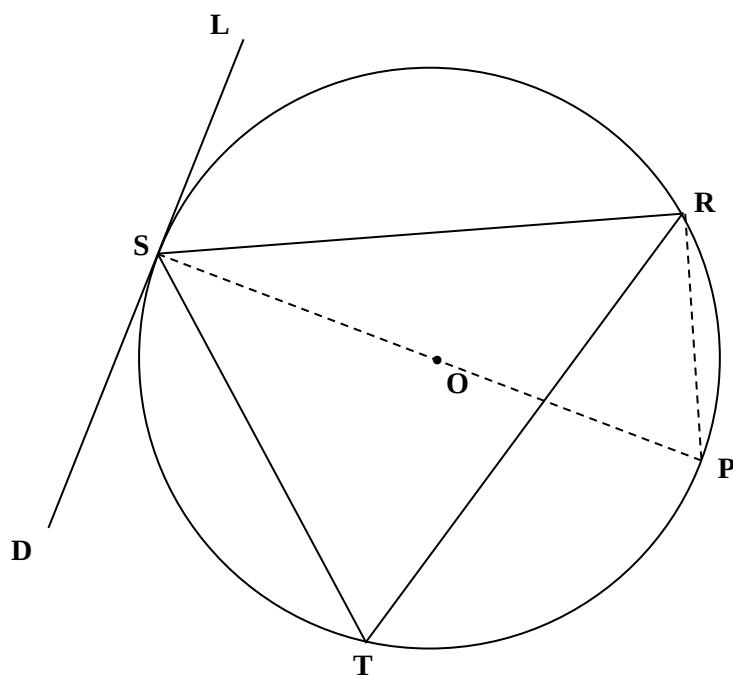


10.1	$\hat{B}_1 = \hat{D}_1 = x$ [∠'e in dieselfde sirkelsegment]	✓ S	
	$\hat{B}_1 = \hat{E}_1 = x$ [∠'e in dieselfde sirkelsegment]	✓ S	(2)
10.2	10.2.1 $\hat{O}_1 = 2x$ [midpunts∠ = 2 × omtreks∠]	✓ S ✓ R	(2)
	10.2.2 $\hat{O}_1 = \hat{O}\hat{F}B + \hat{B}_1$ [buite ∠ van Δ] $2x = \hat{O}\hat{F}B + x$ $\hat{O}\hat{F}B = x$	✓ R ✓ $\hat{O}\hat{F}B = x$	(2)
	10.2.3 $\hat{D}_2 = 90^\circ - x$ [∠'e in 'n halwe sirkel]	✓ S ✓ R	(2)
	10.2.4 $\hat{O}_2 = 180^\circ - 2x$ [∠'e op 'n reguitlyn] $\hat{O}_2 = \hat{E}_2 = 180^\circ - 2x$ [∠'e in dieselfde sirkelsegment]	✓ S ✓ R ✓ S ✓ R	(4)
10.3	$\hat{E}_2 = \hat{B}_3 + \hat{D}_2$ [buite ∠ van Δ] $\therefore \hat{B}_3 = 90^\circ - x$ $\therefore \hat{B}_3 = \hat{D}_2$ [albei $90^\circ - x$]	✓ R ✓ $\hat{B}_3 = 90^\circ - x$ ✓ S	
	$\therefore EB = ED$ [sye teenoor gelyke ∠'e]	✓ R	(4)
			[16]

VRAAG 11

11.1	Konstruksie: Trek middellyn SP en verbind PT	✓ Konstruksie	
	$\hat{L}\hat{S}R = 90^\circ - \hat{R}\hat{S}P$ [rklyn $\perp$ rad]	✓ S ✓ R	
	$\hat{S}\hat{T}R = 90^\circ - \hat{R}\hat{T}P$ [$\angle$ 'e in 'n halwe sirkel]	✓ S ✓ R	
	$\hat{R}\hat{S}P = \hat{R}\hat{T}P$ [$\angle$ 'e in dieselfde sirkelsegment]		
	$\therefore \hat{L}\hat{S}R = \hat{S}\hat{T}R$	✓ S ✓ R	

OF



Konstruksie: Trek middellyn SP en verbind PR

$$\hat{L}SR = 90^\circ - \hat{R}SP \quad [\text{rklyn} \perp \text{rad}]$$

$$\hat{S}RP = 90^\circ \quad [\angle \text{ in 'n halwe sirkel}]$$

$$\hat{S}PR = 90^\circ - \hat{R}SP \quad [\text{binne } \angle \text{'e van } \Delta]$$

$$\hat{L}SR = \hat{S}PR \quad [\text{albei } 90^\circ - \hat{R}SP]$$

$$\hat{S}PR = \hat{S}TR \quad [\angle \text{'e in dieselde segment}]$$

$$\therefore \hat{L}SR = \hat{S}TR$$

✓ Konstruksie

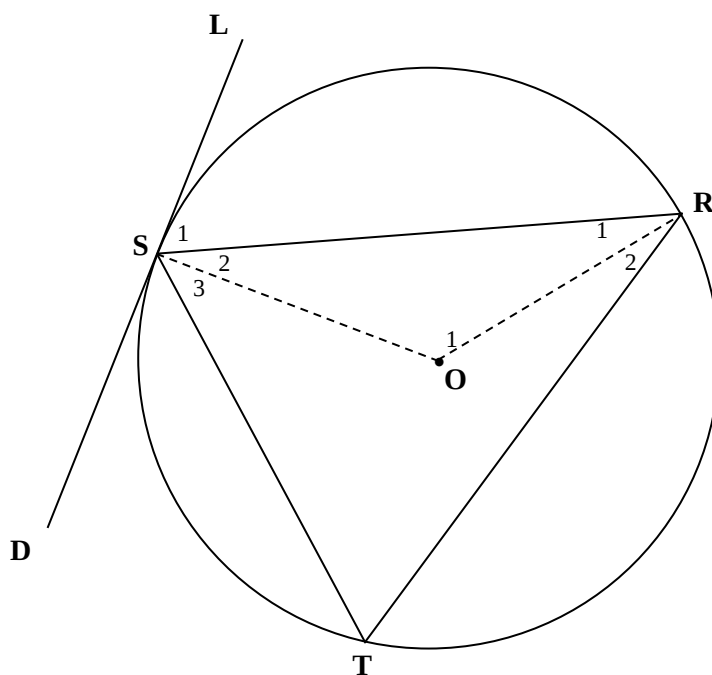
✓ S/R

✓ S ✓ R

✓ S

✓ S/R

Of



Konstruksie: Verbind OS en OR

$$\hat{S}_1 + \hat{S}_2 = 90^\circ \quad [\text{rklyn} \perp \text{radius}]$$

$$\hat{S}_2 = 90^\circ - \hat{S}_1$$

$$\hat{R}_1 = \hat{S}_2 = 90^\circ - \hat{S}_1 \quad [\angle\text{'e teenoor gelyke radii}]$$

$$\hat{O}_1 = 180^\circ - 2[90^\circ - \hat{S}_1] = 2\hat{S}_1 \quad [\text{binne } \angle\text{'e van } \Delta]$$

$$\hat{O}_1 = 2\hat{T} \quad [\text{midpunts}\angle = 2 \times \text{omtreks}\angle]$$

$$\therefore \hat{L}\hat{S}\hat{R} = \hat{S}\hat{T}\hat{R}$$

✓ Konstruksie

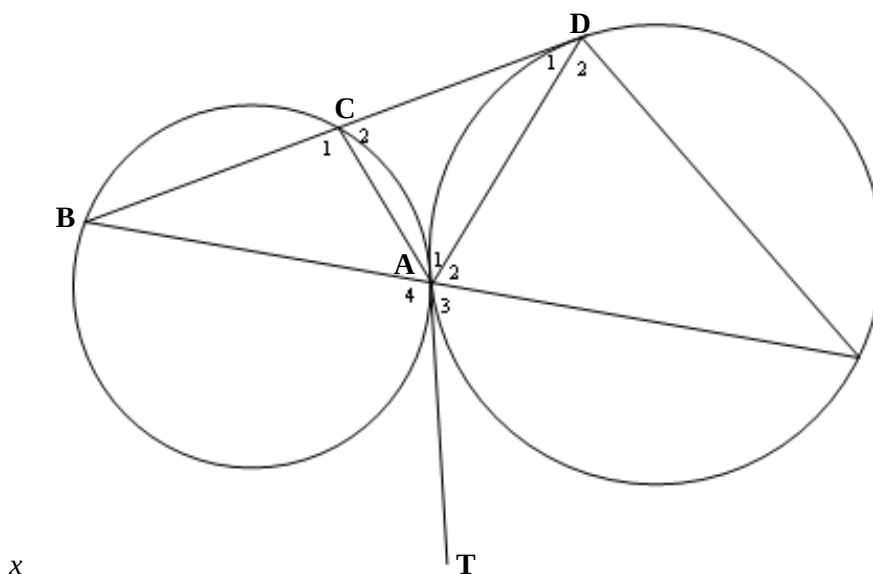
✓ S ✓ R

✓ S/R

✓ S

✓ S ✓ R

(6)

11.2 x

T

	11.2.1	$\angle$ tussen rklyn en koord	✓ R	(1)
	11.2.2	$\hat{A}_4 = 180^\circ - \hat{A}_3$ [$\angle$ 'e op 'n reguitlyn] $\hat{C}_1 = 180^\circ - \hat{A}_3$ [$\angle$ tussen rklyn en koord] $\hat{C}_2 = \hat{A}_3$ [$\angle$ 'e op 'n reguitlyn] $\hat{D}_2 = \hat{A}_3$ [$\angle$ tussen rklyn en koord] $\therefore \hat{C}_2 = \hat{D}_2$ [albei gelyk aan $\hat{A}_3$] $\hat{D}_1 = \hat{F}$ [$\angle$ tussen rklyn en koord /bewys in 11.2.1] $\hat{A}_1 = \hat{A}_2$ [binne $\angle$ 'e van $\Delta/3^{\text{de}}$ $\angle$ in Δ]	✓ SR ✓ S ✓ R ✓ SR ✓ SR ✓ S ✓ R	(7)
				[14]
TOTAAL:				150