



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/GRAAD 11

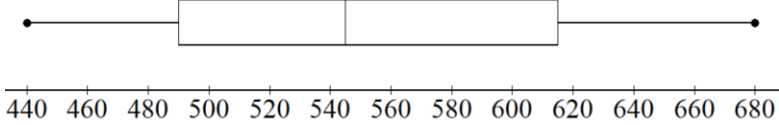
NOVEMBER 2019

**MATHEMATICS P2/WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 13 pages.
Hierdie nasien riglyn bestaan uit 13 bladsye.

QUESTION 1/VRAAG 1

1.1	$\bar{X} = 549,5$	✓✓ answer/antwoord	(2)
1.2	SD = 69,08	✓ answer/antwoord	(1)
1.3	Min/Min : 440 Q ₁ : 490 Q ₂ : 545 Q ₃ : 615 Max/Maks : 680 	✓ Min and max/ Min en maks ✓ Q ₁ ✓ Q ₂ ✓ Q ₃ ✓ box correctly in place / diagram korrek geteken	(5)
1.4	Data skewed slightly right as mean > median	✓✓ slightly skewed to the right/ positively skewed <i>effens skeef na regs / positief skeef</i>	(2)
1.5	Snack bars have greater variety in energy levels as the SD is greater than that of the cereals which means the data is more widely spread about the mean. <i>Peuselstaffies het 'n groter verskeidenheid in energievlakke omdat die SA groter as die van die graanpapsoorte, wat beteken dat die data meer wyd verspreid rondom die gemiddelde is.</i>	✓ snack bars / peuselstaffies ✓ greater SD hence / groter SA ✓ greater spread about the mean / groter verspreiding om die gemiddelde	(3)
			[13]

QUESTION 2/VRAAG 2

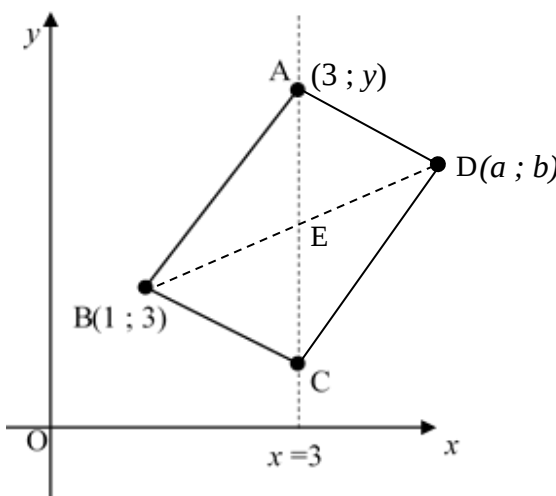
2.1	<table><thead><tr><th>Wind Speed (km/hr) <i>Windsnelheid(km/h)</i></th><th>Frequency <i>Frekwensie</i></th><th>Cumulative Frequency <i>Kumulatiewe Frekwensie</i></th></tr></thead><tbody><tr><td>$10 < x \leq 12$</td><td>1</td><td>1</td></tr><tr><td>$12 < x \leq 14$</td><td>2</td><td>3</td></tr><tr><td>$14 < x \leq 16$</td><td>3</td><td>6</td></tr><tr><td>$16 < x \leq 18$</td><td>4</td><td>10</td></tr><tr><td>$18 < x \leq 20$</td><td>7</td><td>17</td></tr><tr><td>$20 < x \leq 22$</td><td>7</td><td>24</td></tr><tr><td>$22 < x \leq 24$</td><td>4</td><td>28</td></tr><tr><td>$24 < x \leq 26$</td><td>2</td><td>30</td></tr><tr><td>$26 < x \leq 28$</td><td>1</td><td>31</td></tr></tbody></table>	Wind Speed (km/hr) <i>Windsnelheid(km/h)</i>	Frequency <i>Frekwensie</i>	Cumulative Frequency <i>Kumulatiewe Frekwensie</i>	$10 < x \leq 12$	1	1	$12 < x \leq 14$	2	3	$14 < x \leq 16$	3	6	$16 < x \leq 18$	4	10	$18 < x \leq 20$	7	17	$20 < x \leq 22$	7	24	$22 < x \leq 24$	4	28	$24 < x \leq 26$	2	30	$26 < x \leq 28$	1	31	✓ complete freq. column <i>voltooi frekwensie kolom</i> ✓ complete cum. freq. column / <i>voltooi kum. frekwensie kolom</i>	(2)
Wind Speed (km/hr) <i>Windsnelheid(km/h)</i>	Frequency <i>Frekwensie</i>	Cumulative Frequency <i>Kumulatiewe Frekwensie</i>																															
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$24 < x \leq 26$	2	30																															
$26 < x \leq 28$	1	31																															
2.2	Ogive / <i>Ogief</i>	✓ upper boundary values <i>boonste grens waardes</i> ✓ correct points <i>korrekte punte</i> ✓ smooth curve <i>egalige kurwe</i>	(3)																														
2.3.1	Median wind speed = 17,5 km/hr <i>Gemiddelde windsnelheid = 17,5 km/h</i>	✓✓ answer with units <i>antwoord met eenhede</i>	(2)																														
2.3.2	31 – 29 = 2 days/<i>dae</i>	✓ 29 ✓ answer / <i>antwoord</i>	(2)																														
[9]																																	

QUESTION 3 / VRAAG 3

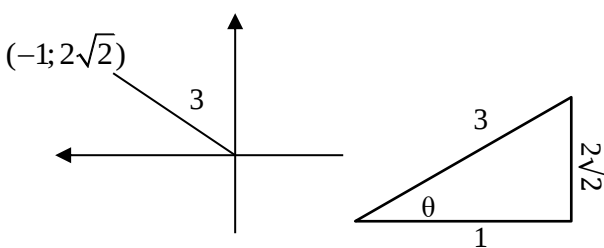
3.1	$3(0) + 2y = 6$ $y = 3$ E (0 ; 3)	✓ substitution / <i>vervanging</i> $x = 0$ ✓ answer / <i>antwoord</i>	(2)
3.2	$y = -\frac{3}{2}x + 3$ $m_{CD} = -\frac{3}{2}$	✓ standard form / <i>standaardvorm</i> ✓ answer / <i>antwoord</i>	(2)
3.3	$\tan^{-1}\left(-\frac{3}{2}\right) = -56.31^\circ$ $\therefore \alpha = 180^\circ - 56.31^\circ$ $\therefore \alpha = 123.69^\circ$ OR Ref. angle / Verw. hoek $= \tan^{-1}\left(\frac{3}{2}\right) = 56.31^\circ$ $\therefore \alpha = 180^\circ - 56.31^\circ$ $\therefore \alpha = 123.69^\circ$	✓ $\tan^{-1}$ ✓ answer / <i>antwoord</i>	(2)
3.4	$\beta = 123.69^\circ - 63.69^\circ$ $\beta = 60^\circ$	✓ answer / <i>antwoord</i>	(1)
3.5	Gradient of/van AB : $\tan(60^\circ) = \sqrt{3}$ E(0 ; 3) $\therefore y = \sqrt{3}x + 3$	✓ use of tan / <i>gebruik van tan</i> ✓✓ answer / <i>antwoord</i>	(3)
3.6	$t = \sqrt{3}(\sqrt{3}) + 3$ $t = 6$	✓ substitute/vervang $x = \sqrt{3}$ ✓ answer / <i>antwoord</i>	(2)

3.7	$0 = \sqrt{3}x + 3$ $\frac{-3}{\sqrt{3}} = x$ $\therefore x = -\sqrt{3}, \text{ hence/vervolgens } A(-\sqrt{3}; 0)$ $3x + 2(0) = 6$ $x = 2, \text{ hence/vervolgens } D(2; 0)$ Length of AD (base of $\triangle ABD$) <i>Lengte van AD (basis van $\triangle ABD$)</i> $= \sqrt{3} + 2 = 3,73$ Height/Hoogte = 6 Hence area $\triangle ABD$ / <i>Vervolgens is oppervlakte van $\triangle ABD$</i> $= \frac{1}{2} \times 3,73 \times 6$ $= 11,19 \text{ units}^2/\text{eenhede}^2$ OR Length of AD (base of $\triangle ABD$) <i>Lengte van AD (basis van $\triangle ABD$)</i> $= \sqrt{3} + 2$ Hence area $\triangle ABD$ / <i>Vervolgens is oppervlakte van $\triangle ABD$</i> $= \frac{1}{2} \times (\sqrt{3} + 2) \times 6$ $= 11,20 \text{ units}^2$	✓ calculation of A / <i>berekening van A</i> ✓ calculation of D / <i>berekening van D</i> ✓ height / hoogte ✓ area formula / <i>oppervlakte formule</i> ✓ answer / antwoord	(5)
			[17]

QUESTION 4 / VRAAG 4

4.1	 $(2\sqrt{5})^2 = (3-1)^2 + (y-3)^2$ $20 = 4 + (y-3)^2$ $16 = (y-3)^2$ OR/OF $20 = 4 + y^2 - 6y + 9$ $3 \pm 4 = y$ OR/OF $0 = y^2 - 6y - 7$ $y = 7 \text{ or } -1$ OR/OF $0 = (y-7)(y+1)$ $A(3; 7)$	✓ distance formula <i>afstand formule</i> ✓ $x = 3$ and/en $(y - 3)$ ✓ solving for y <i>oplossing vir y</i> ✓ coordinates <i>koördinate</i>	(4)
4.2	C(3; 2)	✓✓ coordinates <i>koördinate</i>	(2)
4.3	$\left(\frac{3+3}{2}; \frac{7+2}{2}\right) = (3; 4,5)$	✓✓ substitution in corr formula <i>vervanging in korrekte formule</i>	(2)
4.4	$\left(\frac{1+a}{2}; \frac{3+b}{2}\right) = (3; 4,5)$ $\frac{1+a}{2} = 3 \Rightarrow 1+a = 6 \Rightarrow a = 5$ $\frac{3+b}{2} = 4,5 \Rightarrow 3+b = 9 \Rightarrow b = 6$ D(5;6) OR/OF using the gradient of AB = $\frac{4}{2}$ = gradient of BC Hence C(3;2) moved up 4 and across 2 $\Rightarrow$ D(5;6)	✓ use of midpoint theorem <i>Gebruik van middelpunt stelling</i> ✓✓ coordinates of D <i>koördinate van D</i> ✓ gradient/<i>gradiënt</i> ✓✓ coordinates of D	(3)
4.5	B'(9;0)		(2)
[13]			

QUESTION 5/VRAAG 5

5.1		✓ diagram / <i>diagram</i>	
5.1.1	$\tan(180^\circ + \theta)$ $= \tan \theta$ $= -2\sqrt{2}$	✓ reduction / <i>reduksie</i> ✓ answer / <i>antwoord</i>	(3)
5.1.2	$3\sin(\theta - 90^\circ)$ $= 3(-\cos \theta)$ $= 3\left(-\frac{1}{3}\right)$ $= -1$	✓ reduction / <i>reduksie</i> ✓ answer / <i>antwoord</i>	(2)
5.2.1	$\frac{\sin(-210^\circ)}{\cos(300^\circ)} + \frac{\cos(x+90^\circ)}{\sin(360^\circ+x)}$ $\frac{\sin 30^\circ}{\cos 60^\circ} + \frac{-\sin x}{\sin x}$ $\frac{1}{\frac{1}{2}} + -1$ $= 2 + -1$ $= 1$ $= 0$	✓✓ sin and cos reduction sin en cos <i>reduksie</i> ✓✓ cos and sin reduction cos en sin <i>reduksie</i> ✓ answer / <i>antwoord</i>	(5)
5.2.2	$\sin(360^\circ + x) \neq 0$ $360^\circ + x \neq 0^\circ + k \cdot 360^\circ$ or/of $360^\circ + x \neq 180^\circ + k \cdot 360^\circ$ $x \neq -360^\circ; -180^\circ; 0; 180^\circ; 360^\circ$	✓✓ answers / <i>antwoorde</i>	(2)
5.3	$\text{LHS/LK} : \tan \theta \sqrt{\frac{1}{\sin^2 \theta} - 1}$ $= \frac{\sin \theta}{\cos \theta} \sqrt{\frac{1 - \sin^2 \theta}{\sin^2 \theta}}$ $= \frac{\sin \theta}{\cos \theta} \sqrt{\frac{\cos^2 \theta}{\sin^2 \theta}}$ $= \frac{\sin \theta}{\cos \theta} \times \frac{\cos \theta}{\sin \theta}$ $= 1$ $= \text{RHS/RK}$	✓ single fraction/enkel <i>breuk</i> ✓ changing tan/ <i>verander tan</i> ✓ identity / <i>identiteit</i> ✓ taking square root / <i>vierkantwortel</i>	(4)

5.4	$2\sin^2 \theta = 1 + \sin \theta$ $2\sin^2 \theta - \sin \theta - 1 = 0$ $(2\sin \theta + 1)(\sin \theta - 1) = 0$ $\sin \theta = -\frac{1}{2}$ or/of $\sin \theta = 1$ $\therefore \theta = -30^\circ + k.360^\circ$ or/of $\theta = 210^\circ + k.360^\circ$ OR/OF $\theta = 330^\circ + k.360^\circ$ or/of $\theta = 210^\circ + k.360^\circ$ or/of $\theta = 90^\circ + k.360^\circ$; $k \in \mathbb{Z}$	✓ standard form / <i>standaardvorm</i> ✓ factorising / <i>faktorisering</i> ✓ solving / <i>los op</i> ✓✓✓ solutions / <i>oplossings</i> Penalise 1 mark if no $k \in \mathbb{Z}$ <i>Penaliseer 1 punt indien</i> <i>geen $k \in \mathbb{Z}$</i>	(6)
			[22]

QUESTION 6 / VRAAG 6

6.1		<i>f</i> ✓ turning pts/draaipunte ✓ x-intercepts/x-afsnitte ✓ y-intercept / y-afsnit <i>g:</i> ✓ intercepts / afsnitte ✓ turning pts/draaipunte ✓ shape / vorm	(6)
6.2.1	$x = 0^\circ$	✓ answer/antwoord	(1)
6.2.2	$x \in [-90^\circ; 90^\circ]$ OR/OF $-90^\circ \leq x \leq 90^\circ$	✓✓ interval end points / interval eindpunte	(2)
6.2.3	$x \in (0^\circ; 45^\circ)$ OR/OF $0^\circ < x < 45^\circ$ $x \in (135^\circ; 180^\circ)$ OR/OF $135^\circ < x < 180^\circ$	✓ answer/antwoord ✓ answer/antwoord	(2)
6.3	$h(x) = 2\cos(x - 30^\circ)$	✓ positive cos / positiewe cos ✓ $(x - 30^\circ)$	(2)
[13]			

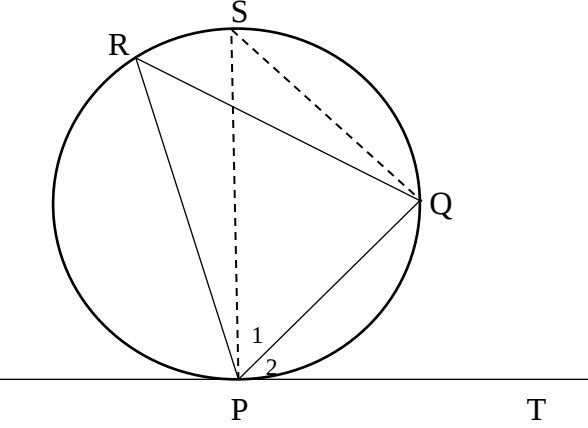
QUESTION 7 / VRAAG 7

7.1.1	$\widehat{BDC} = 70^\circ$	✓ answer / antwoord	(1)
7.1.2	$\frac{BD}{\sin 50^\circ} = \frac{5}{\sin 20^\circ}$ $BD = \frac{5 \sin 50^\circ}{\sin 20^\circ}$ $BD = 11,20 \text{ m}$ $\sin 70^\circ = \frac{BC}{11,2}$ $BC = 11,2 \times \sin 70^\circ$ $BC = 10,52 \text{ m}$	✓ sine rule / sinusreël ✓ correct substitution / korrekte vervanging ✓ answer for AC / antwoord vir AC ✓ sine ratio/sinus verhouding ✓ answer / antwoord	(5)
7.2.1	$AB^2 = r^2 + r^2 - 2 \times r \times r \times \cos 45^\circ$ $AB^2 = 2r^2(1 - \cos 45^\circ)$ $AB^2 = r^2 \left(2 - 2 \times \frac{\sqrt{2}}{2}\right)$ $AB = r(\sqrt{2} - \sqrt{2})$ $\text{Perimeter/Omtrek} = 8 \times AB = 8r\sqrt{2} - \sqrt{2}$	✓ use of cos rule gebruik van cos-reël ✓ expression for AB uitdrukking vir AB ✓ answer / antwoord	(3)
7.2.2	Area of the octagon / Oppervlakte van agthoek $= 8 \times \text{area } \triangle AOB$ $= 8 \times \frac{1}{2} \times r \times r \times \sin 45^\circ$ $= 8 \times \frac{1}{2} \times r \times r \times \frac{\sqrt{2}}{2}$ $= 2\sqrt{2}r^2$	✓ ×8 ✓ angle in triangle/hoek in driehoek ✓ area formula/oppervlakte formule ✓ sin 45° value / waarde	(4)
			[13]

QUESTION 8 / VRAAG 8

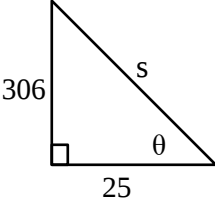
8.1.1	$\hat{A} = 45^\circ$ ($\angle$ at centre = $2 \times \angle$ at circum.) <i>(Middelpunts $\angle = 2 \times$ Omtreks $\angle$)</i> $\hat{D} = 45^\circ$ ($\angle$ s in the same seg./ $\angle$ e in dieselfde segm.) $\hat{ACD} = 45^\circ$ OR/OF $\hat{ABD} = 45^\circ$ (<i>alt $\angle$'s $\parallel$ lines</i>)	$\checkmark$ (S) and (R) (S) en (R) $\checkmark$ S $\checkmark$ R	(3)
8.1.2 (a)	R.T.P /Te Bewys: $CE = DE$ Proof/Bewys : $\hat{ACD} = \hat{D}$ (proved above / <i>bo bewys</i>) $\therefore CE = DE$ (equal sides opp equal $\angle$'s) <i>(gelyke sye teenoor gelyke hoeke)</i>	$\checkmark$ S $\checkmark$ R	(2)
8.1.2 (b)	R.T.P/Te Bewys : CD is the diameter of a circle passing through ECD / <i>CD is die middellyn van die sirkel deur punte E, C en D gaan</i> Proof/Bewys : $\hat{E} = 90^\circ$ (3 $\angle$'s/e Δ) $\therefore$ CD is diameter (line subtends 90°) <i>CD is die middellyn (lyn onderspan 90°)</i>	$\checkmark$ S $\checkmark$ R	(2)
8.2.1 (a)	$\hat{C}_1 = 70^\circ$ (equal $\angle$'s opp equal sides / <i>gelyke $\angle$e teenoor gelyke sye</i>)	$\checkmark$ S $\checkmark$ R	(2)
8.2.1 (b)	$\hat{A}_2 = 35^\circ$ (exterior $\angle$ of Δ / <i>buitehoek van driehoek</i>)	$\checkmark$ S $\checkmark$ R	(2)
8.2.1 (c)	$\hat{B}_2 = 35^\circ$ ($\angle$'s in same segment / <i>$\angle$e in dieselde segment</i>)	$\checkmark$ S $\checkmark$ R	(2)
8.2.2	R.T.P./Te Bewys : $\hat{B}_1 = \hat{B}_2$ Proof/Bewys : $\hat{B}_2 = 35^\circ$ (proved above / <i>bo bewys</i>) $\hat{ABC} = \hat{C}_1 = 70^\circ$ (equal $\angle$'s opp equal sides / <i>gelyke $\angle$e teenoor gelyke sye</i>) $\therefore \hat{B}_1 = 70^\circ - 35^\circ$ $\therefore \hat{B}_1 = \hat{B}_2$ $\therefore$ BE bisects $\hat{ABC}$ / <i>BE halveer $\hat{ABC}$</i>	$\checkmark$ S $\checkmark$ R $\checkmark \hat{B}_1 = \hat{B}_2$	(3)
8.3.1	R.T.P/Te Bewys : CDOB is a cyclic quadrilateral / <i>is 'n koordevierhoek</i> Proof/Bewys : $\hat{O}_3 = 90^\circ$ ($CO \perp AB$ given / <i>gegee</i>) $\hat{ADB} = 90^\circ$ (angle in a semi-circle)/(<i>hoek in halwe sirkel</i>) $\hat{D}_3 = 90^\circ$ (angles on st line) / (<i>hoeke op 'n reguitlyn</i>) $\therefore$ CDOB is a cyclic quadrilateral / <i>is 'n koordevierhoek</i> <i>($\angle$'s in same seg)</i>	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S $\checkmark$ S	(4)
8.3.2	R.T.P/Te Bewys. : $\hat{D}_2 = \hat{C}_1$ Proof/Bewys : $\hat{D}_2 = \hat{B}_1 = x$ (equal $\angle$'s opp equal radii)/ <i>(gelyke $\angle$e teenoor gelyke radiusse)</i> $\hat{B}_1 = \hat{C}_1 = x$ ($\angle$'s in the same segment of cyclic quad.) <i>($\angle$e in dieselfde segment van koordevierhoek)</i> $\therefore \hat{D}_2 = \hat{C}_1$	$\checkmark$ S/R $\checkmark$ S/R	(2)
[22]			

QUESTION 9 / VRAAG 9

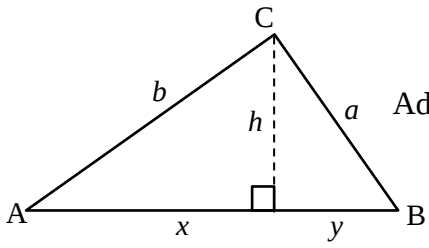
9.1	 R.T.P/Te Bewys : $\hat{P}_2 = \hat{R}$ Construction: Draw diameter SP. Join SQ Konstruksie: Teken middellyn SP. Verbind SQ Proof/Bewys : $\hat{P}_1 + \hat{P}_2 = 90^\circ$ (tang $\perp$ diameter)/ (raaklyn $\perp$ middellyn) $\hat{SQP} = 90^\circ$ ($\angle$ in semi-circle)/($\angle$ in halwe sirkel) $\hat{P}_1 + \hat{S} = 90^\circ$ ($\angle$'s in Δ) / ($\angle$e in Δ) But/Maar $\hat{S} = \hat{R}$ ($\angle$'s in same segment) / ($\angle$e in dieselfde segment) $\therefore \hat{P}_1 + \hat{R} = \hat{P}_1 + \hat{P}_2 = 90^\circ$ Hence/Vervolgens is $\hat{R} = \hat{P}_2$	✓ construction/ konstruksie ✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)
9.2.1	$\hat{B}_2 = y$ (tan chord thrm) / (raaklyn koord stelling)	✓ S ✓ R	(2)
9.2.2	$\hat{D}_3 = x + y$ (ext $\angle$ of Δ) / (buite $\angle$ van Δ)	✓ S ✓ R	(2)
9.3	R.T.P/Te Bewys : $AB \parallel EC$ Proof/Bewys : $\hat{C}_2 = \hat{D}_3 = x + y$ ($\angle$'s in same segment) ($\angle$ 'e in dieselfde segment) $\hat{BAC} = x + y$ $\therefore AB \parallel EC$ (equal alt $\angle$'s) / (gelyke verw. $\angle$ e)	✓ S ✓ R ✓ R	(3)
9.4.1	R.T.P/Te Bewys : $\hat{C}_3 = \hat{F}$ Proof/Bewys : $\hat{C}_3 = \hat{D}_1$ ($\angle$'s in same segment) / ($\angle$ 'e in dieselfde segment) $\hat{D}_1 = \hat{F}$ (corresp $\angle$'s $\parallel$ lines) / (ooreenk. $\angle$: $\parallel$ lyne) $\therefore \hat{C}_3 = \hat{F}$	✓ S ✓ R ✓ S	(3)

9.4.2	R.T.P / <i>Te Bewys</i> : $\hat{ACF} = \hat{CDF}$ Proof / <i>Bewys</i> : $\hat{ACF} = 180^\circ - (\hat{A}_1 + \hat{F})$ ($\angle$'s of Δ) / ($\angle$e van Δ) $\hat{CDF} = 180^\circ - (\hat{C}_1 + \hat{F})$ ($\angle$'s of Δ) / ($\angle$e van Δ) But $\hat{C}_1 = \hat{A}_1$ (tan chord thrm) / (<i>raaklyn koord stelling</i>) $\therefore \hat{ACF} = \hat{CDF}$ OR/OF $\hat{ACF} = \hat{C}_1 + \hat{C}_2$ and/en $\hat{C}_1 = \hat{B}_2$ (tan chord thrm) / (<i>raaklyn koord stelling</i>) $\hat{C}_2 = \hat{B}_1$ ($\angle$'s in same segment) / ($\angle$e in dies. segment) $\hat{CDF} = \hat{B}_1 + \hat{B}_2$ (ext $\angle$ cyclic quad) / (<i>buite $\angle$ van k.v</i>)	✓ S ✓ S ✓ R ✓ S ✓ S ✓ R	(3)
			[18]

QUESTION 10 / VRAAG 10

10.1	Surface area of sides = $\frac{1}{2} \times$ perimeter of the base $\times$ slant ht <i>Buite oppervlakte van sye = $\frac{1}{2} \times$ omtrek van basis $\times$ skuinshg</i> Slant height/Skuinshoogte: $306^2 = 25^2 + (s)^2$ Pythagoras $s = 304,98$ m  $SA = \frac{1}{2} \times 4(50) \times 304,98$ $= 30498 \text{ m}^2$	✓ Pytha eqn Pyth. verg. ✓ 25m ✓ substitution in formula vervanging in formule ✓ answer / antwoord	(4)
10.2	$\tan^{-1} \theta = \frac{306}{25}$ $\theta = 85,33^\circ$	✓ arctan ✓ answer/ antwoord	(2)
			[6]

QUESTION 11 / VRAAG 11

 Hence triangle is rht $\angle$'d given : $h^2 = xy$ <i>Die driehoek is vervolgens reghoekig, gegee dat: $h^2 = xy$</i>	$a^2 = h^2 + y^2$ $b^2 = h^2 + x^2$ Adding/Optel : $a^2 + b^2 = 2h^2 + x^2 + y^2$ $= 2(xy) + x^2 + y^2$ $a^2 + b^2 = (x + y)^2$ $BC^2 + AC^2 = AB^2$	✓ use of Pythag <i>gebruik van Pyth.</i> ✓ adding / optel ✓ substitution / vervanging ✓ factorising / faktorisering	[4]
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TOTAL/TOTAAL: 150