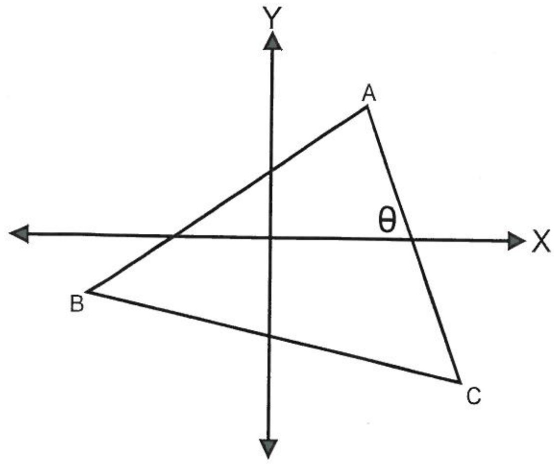
(1) A(2 ; 4) , B(-4 ; -2) and C(4 ; -4) is the vertices of Δ ABC.



(a) Determine the coordinates of P and Q if P and Q are the midpoints of respectively AB and AC.

(b) Prove that PQ // BC.

(c) Prove that $PQ = \frac{1}{2}BC$.

(d) Calculate the size of θ correct to the nearest degree.

(a) $A(2;4)$ and $B(-4;-2)$

$$mP_{AB} = \left(\frac{2-4}{2} ; \frac{4-2}{2}\right)$$
$$= (-1;1)$$

$A(2;4)$ and $C(4;-4)$

$$mP_{AC} = \left(\frac{2+4}{2} ; \frac{4-4}{2}\right)$$
$$= (3;0)$$

(b) $m_{PQ} = \frac{0-1}{3-(-1)}$ $\qquad$ $P(-1;1)$ and $Q(3;0)$

$$\therefore m_{PQ} = \frac{-1}{4}$$

$m_{BC} = \frac{-4-(-2)}{4-(-4)}$ $\qquad$ $B(-4;-2)$ and $C(4;-4)$

$$= \frac{-2}{8} = -\frac{-1}{4}$$

$$\therefore m_{PQ} = m_{BC}$$

$$\therefore PQ \parallel BC$$

(c) $P(-1;1)$ and $Q(3;0)$

$d(PQ) = \sqrt{(-1-3)^2 + (1-0)^2}$

$= \sqrt{(-4)^2 + (1)^2}$

$= \sqrt{16+1}$

$d(PQ) = \sqrt{17}$

$B(-4;-2)$ and $C(4;-4)$

$d(BC) = \sqrt{(-4-4)^2 + (-2+4)^2}$

$= \sqrt{(-8)^2 + (2)^2}$

$= \sqrt{64+4}$

$d(BC) = \sqrt{68}$

but $\sqrt{68} = 2\sqrt{17}$

$\therefore BC = 2PQ$

$\therefore PQ = \frac{1}{2}BC$

(d) $\tan(180° - \theta) = m_{AC}$ $\quad$ $A(2;4)$ en $C(4;-4)$

$\therefore \tan(180° - \theta) = -4$ $\quad$ $m_{AC} = \frac{-4-4}{4-2}$

$-\tan\theta = -4$ $\quad$ $= \frac{-8}{2}$

$\tan\theta = 4$ $\quad$ $m_{AC} = -4$

$\therefore \theta = 75{,}96\ldots$

$\therefore \theta \approx 76°$

(2) P(0 ; 2) , Q(2 ; 5) , R(-1 ; 3) and S is vertices of parallelogram PQRS.

(a) Calculate the gradient of line QR.

(b) Calculate the equation of line PS.

(c) Prove that PQRS is a rhombus.

(d) Calculate the coordinates of the point of intersection of the diagonals of PQRS.

(a) $Q(2;5)$ and $R(-1;3)$

$\therefore m_{QR} = \frac{3-5}{-1-2} = \frac{-2}{-3} = \frac{2}{3}$

(b) $PS \parallel QR$ [opp. sides of Parm]

$\therefore m_{PS} = m_{QR} = \frac{2}{3}$

$\therefore y = mx + c$ through $P(0;2)$ c

$y = \frac{2}{3}x + 2$

(c) $d(PQ) = \sqrt{(0-2)^2 + (2-5)^2}$ $P(0;2)$ and $Q(2;5)$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$d(QR) = \sqrt{(2-(-1))^2 + (5-3)^2}$ $Q(2;5)$ and $R(-1;3)$

$= \sqrt{(3)^2 + (2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

$\therefore$ PQRS is a rhombus $\rightarrow$ adjacent sides =

(d) Intersection of diagonals is the midpoint of PR (or QS, but S is unknown.)

$P(0;2)$ $R(-1;3)$

$\therefore \ mp = \left(\frac{0-1}{2} ; \frac{2+3}{2}\right)$

$= \left(\frac{-1}{2} ; \frac{5}{2}\right)$

(3) TWK is an isosceles triangle with TW = WK. T(7 ; 8) , W(1 ; 6) and K(-1 ; y).

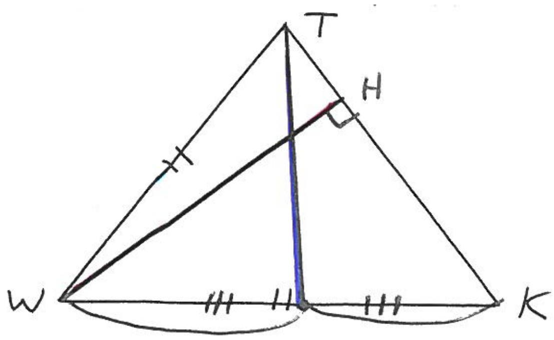
(a) Calculate the length of TW.

(b) Calculate the value(s) of y.

(c) Calculate the equation of the:

(i) median through T for $y > 0$.

(ii) altitude through W for $y \leq 0$.



(a) T(7;8) and W(1;6)

$d(TW) = \sqrt{(7-1)^2 + (8-6)^2}$

$= \sqrt{(6)^2 + (2)^2}$

$= \sqrt{36 + 4}$

$\therefore d(TW) = \sqrt{40}$

(b) W(1;6) and K(-1;y)

$d(WK) = \sqrt{(1-(-1))^2 + (6-y)^2}$

$\sqrt{40} = \sqrt{(1+1)^2 + 36 - 12y + y^2}$

$(\sqrt{40})^2 = (\sqrt{(2)^2 + 36 - 12y + y^2})^2$

$40 = 4 + 36 - 12y + y^2$

$0 = y^2 - 12y + 40 - 40$

$0 = y^2 - 12y$

$0 = y(y-12)$

$y = 0$ or $y = 12$

(c)(i) median through T → to mp of WK

W(1;6) and K(−1;12) $y>0$

$\therefore$ $mp_{WK} = \left(\frac{1-1}{2}; \frac{6+12}{2}\right)$

$= (0;9)$

$\therefore$ line through T(7;8) and (0;9) → c

$m = \frac{9-8}{0-7} = \frac{1}{-7} = -\frac{1}{7}$

$\therefore$ $y = mx + c$

$y = -\frac{1}{7}x + 9$

(ii) altitude through W → WH ⊥ TK

T(7;8) and K(−1;0) $y \leq 0$

$\therefore$ $m_{TK} = \frac{0-8}{-1-7} = \frac{-8}{-8} = 1$

$\therefore$ $m_{WH} = -1$ $(m_{TK} \times m_{WH} = -1)$

$\therefore$ $y - y_1 = m(x - x_1)$ through W(1;6)

$y - 6 = -1(x-1)$

$y - 6 = -1x + 1$

$y = -1x + 7$

$y = -x + 7$