MEMO 2023-12ƐD-00012(1) GS Applications

(1) A virus spreads at a rapid pace. On the first day, 4 000 people were infected by the virus. Each consecutive day the number of infections doubled. How many infections were recorded within a week?

Sequence: $4\,000 + 8\,000 + 16\,000 + \ldots$

$a = 4\,000 \qquad S_n = \frac{a(r^n - 1)}{r - 1}$

$r = 2 \qquad \therefore \quad S_7 = \frac{4\,000(2^7 - 1)}{2 - 1}$

$n = 7 \qquad \therefore \quad S_7 = \frac{4\,000(2^7 - 1)}{1}$

$S_n = \ ? \qquad \therefore \quad S_7 = 508\,000$

∴ After one week (7 days) a total of 508 000 infections will have been recorded.

(2) The sum of the areas of a series of squares is calculated. The first square has a side length of 16 cm. Each consecutive square's side length is half of the previous square's side length. Write down the areas of the first five squares.

(a) The sum of the areas of a series of squares is calculated. The first square has a side length

$16^2 \ ; \ 8^2 \ ; \ 4^2 \ ; \ 2^2 \ ; \ 1^2 \ = \ 256 \ ; \ 64 \ ; \ 16 \ ; \ 4 \ ; \ 1$

(b) Calculate the area of the 15^{th} square in the series.

$a = 256 \qquad T_n = a.r^{n-1}$

$r = \frac{1}{4} \qquad \therefore \quad T_{15} = 256 \,.\left(\frac{1}{4}\right)^{15 - 1}$

$n = 15 \qquad \therefore \quad T_{15} = 256 \times (0,25)^{14}$

$T_n = ? \qquad \therefore \quad T_{15} = \frac{1}{1\,048\,576}\ \text{cm}^2$

∴ The area of the 15^{th} square will be $\frac{1}{1\,048\,576}$ cm^2.

(c) Calculate the sum areas of the first 10 squares. [Correct to 2 decimals.]

$a = 256 \qquad S_n = \frac{a(1 - r^n)}{1 - r}$

$r = \frac{1}{4} \qquad \therefore \quad S_{10} = \frac{256(1 - 0{,}25^n)}{1 - 0{,}25}$

$n = 10 \qquad \therefore \quad S_{10} = \frac{256(1 - 0{,}25^{10})}{0{,}75}$

$S_n = ? \qquad \therefore \quad S_{10} \approx 341{,}33$

∴ The sum of the areas of the first 10 squares is $341{,}33$ cm^2.

(d) Calculate the maximum total area of all the squares in the series.

This series converges $\rightarrow$ $\therefore$ the maximum area will be S_∞.

$$a = 256 \qquad\qquad S_\infty = \frac{a}{1-r}$$

$$r = \frac{1}{2} \qquad \therefore \qquad S_\infty = \frac{256}{1-\frac{1}{2}}$$

$$S_\infty = ? \qquad \therefore \qquad S_\infty = 512\ \text{cm}^2$$

$\therefore$ The maximum total area is $512\ \text{cm}^2$.