Mathematics Gr.11 June Paper 1 (0001)

Time: 2H **Total :100 marks**

Question 1: [14]

Solve for x. Where necessary, round off to 2 decimals:

1.1 $(x + 1)(2x - 1) = 0$ **(2)**

$x+1=0$ or $2x-1=0$

$x=-1$ ✓ $x=\frac{1}{2}$ ✓

1.2 $(2x - 1)(x + 4) = -4$ **(4)**

$2x^2+8x-x-4=-4$ ✓

$2x^2+7x-4+4=0$

$2x^2+7x=0$ ✓

$x(2x+7)=0$

$x=0$ ✓ or $x=-\frac{7}{2}$ ✓

1.3 $(x^2 - 2x)^2 = 9$ **(6)**

$x^2-2x=\pm 3$

$x^2-2x=3$ or $x^2-2x=-3$

$x^2-2x-3=0$ ✓ $x^2-2x+3=0$ ✓

$(x-3)(x+1)=0$ $x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(3)}}{2(1)}$

$x=3$ ✓ or $x=-1$ ✓ $x=\frac{2\pm\sqrt{-8}}{2}$ ✓

No R ✓ solution

1.4 $4x^2 + 9 = 0$ **(2)**

$4x^2=-9$ $x=\pm\sqrt{\frac{-9}{4}}$ ✓

$x^2=-\frac{9}{4}$ No R ✓ solution

Question 2: [7]

Solve for y:

$$\frac{y}{y+1} + \frac{2y}{y-1} - \frac{y^2+3}{y^2-1} = \frac{9}{4}$$

$$\frac{y}{(y+1)} + \frac{2y}{(y-1)} - \frac{(y^2+3)}{(y-1)(y+1)} = \frac{9}{4}$$

$LCD = 4(y+1)(y-1)$

$\therefore\ y \neq -1 \quad$ or $\quad y \neq 1$

$\therefore 4y(y-1) + 4 \times 2y(y+1) - 4(y^2+3) = 9(y-1)(y+1)$

$4y^2 - 4y + 8y^2 + 8y - 4y^2 - 12 = 9(y^2-1)$

$8y^2 + 4y - 12 = 9y^2 - 9$

$8y^2 + 4y - 12 - 9y^2 + 9 = 0$

$-y^2 + 4y - 3 = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y - 3 = 0 \quad$ or $\quad y - 1 = 0$

$y = 3$ $\qquad$ $y = 1$

N/A

Question 3: [10]

3.1 Solve for k: $k^2 - k - 20 = 0$ **(3)**

$(k-5)(k+4) = 0$

$k-5=0$ or $k+4=0$

$k=5$ ✓ $k=-4$ ✓

3.2 Use you answer in 3.1 and solve for x:

$(x^2 + 4x)^2 - (x^2 + 4x) - 20 = 0$ **(7)**

Let $x^2+4x = k$ ✓

$\therefore \quad k^2 - k - 20 = 0$

$k=5$ or ✓ $k=-4$ [See 3.1]

$\therefore \quad x^2+4x = 5$ ✓ $x^2+4x = -4$ ✓

$x^2+4x-5=0$ $x^2+4x+4=0$

$(x+5)(x-1)=0$ $(x+2)(x+2)=0$

$x=-5$ ✓ or ✓ $x=1$ $x=-2$ or ✓ $x=-2$

Question 4: [8]

Solve for x. Remember to check your answers!

$$x + \sqrt{2x - 5} = 2$$

$\sqrt{2x-5} = 2-x$ ✓

$(\sqrt{2x-5})^2 = (2-x)^2$ ✓

$2x - 5 = 4 - 4x + x^2$ ✓

$0 = x^2 - 4x + 4 - 2x + 5$

$0 = x^2 - 6x + 9$ ✓

$0 = (x-3)(x-3)$

$\therefore x = 3$ ✓

Check: LHS $= x + \sqrt{2x-5}$ RHS $= 2$

$= 3 + \sqrt{2\times3-5}$

$= 3 + 1 = 4$ $\therefore$ LHS $\neq$ RHS

$\therefore x \neq 3$ ✓

$\therefore$ No solution ✓

Question 5: [8]

Solve for x and y:

$$x^2 + 2xy = 3y^2 - 7 \quad \text{and} \quad 3x - y = 1$$

$3x - 1 = y$ ✓ ←

$x^2 + 2x(3x-1) = 3(3x-1)^2 - 7$ ✓

$x^2 + 6x^2 - 2x = 3(9x^2 - 6x + 1) - 7$

$7x^2 - 2x = 27x^2 - 18x + 3 - 7$

$0 = 27x^2 - 18x - 4 - 7x^2 + 2x$

$0 = 20x^2 - 16x - 4$ ✓

$0 = 5x^2 - 4x - 1$

$0 = (5x+1)(x-1)$

$x = -\frac{1}{5}$ ✓ or $x = 1$ ✓

$\therefore$ $y = 3(-\frac{1}{5}) - 1$ $y = 3(1) - 1$

$y = -\frac{8}{5}$ ✓ $y = 2$ ✓

Question 6: [15]

No calculator may be used in this question!

6.1 Simplify: (a) $\dfrac{12^{x+1} \times 9^{x-2} \times 3^{x}}{18^{2x-1}}$ **(5)**

$= \dfrac{(2^2 \times 3)^{x+1} \times (3^2)^{x-2} \times 3^{x}}{(3^2 \times 2)^{2x-1}}$

$= \dfrac{2^{2x+2} \times 3^{x+1} \times 3^{2x-4} \times 3^{x}}{3^{4x-2} \times 2^{2x-1}}$

$= \dfrac{2^{2x+2} \times 3^{4x-3}}{2^{2x-1} \times 3^{4x-2}}$

$= 2^{2x+2-(2x-1)} \times 3^{4x-3-(4x-2)}$

$= 2^{2x+2-2x+1} \times 3^{4x-3-4x+2}$

$= 2^{3} \cdot 3^{-1}$

$= \dfrac{8}{3}$

(b) $\sqrt{27} - \sqrt{3} - \sqrt{12}$ **(3)**

$= 3\sqrt{3} - 1\sqrt{3} - 2\sqrt{3}$

$= 0$

6.2 Solve for x: (a) $2x^{-\frac{3}{2}} = 54$ **(3)**

$x^{-\frac{3}{2}} = \frac{54}{2} = 27$

$(x^{-\frac{3}{2}})^{-\frac{2}{3}} = (3^3)^{-\frac{2}{3}}$

$x^{1} = 3^{-2}$

$x = \frac{1}{3^2} = \frac{1}{9}$

(b) $2^{3x+1} + 2^{3x} = 12$ (4)

$2^{3x} \cdot 2^1 + 2^{3x} = 12$

$2^{3x}(2+1) = 12$

$2^{3x}(3) = 12$

$2^{3x} = \frac{12}{3}$

$2^{3x} = 4$

$2^{3x} = 2^2$

$\therefore 3x = 2$

$x = \frac{2}{3}$

Question 7:[10]

The second difference in a quadratic sequence is 2.
The first two terms of the sequence is 2 and 9.

7.1 Determine the n^{th} term of the sequence. **(5)**

2 ; 9 ; .....

7

with 2nd difference = 2

$\circledast \therefore 2a = 2$

$a = 1$

$\circledast \; 3a + b = 7$

$3(1) + b = 7$

$b = 4$

$\circledast \; a + b + c = 2$

$1 + 4 + c = 2$

$c = -3$

$\therefore T_n = n^2 + 4n - 3$

7.2 Which term of the sequence will be equal to 218? **(5)**

$T_n = n^2 + 4n - 3$ ✓

$218 = n^2 + 4n - 3$

$0 = n^2 + 4n - 3 - 218$

$0 = n^2 + 4n - 221$ ✓

$0 = (n + 17)(n - 13)$

$n = -17$ ✓ or $n = 13$ ✓

N/A ✓

Question 8:[7]

8.1 Sketch the following graphs on the same systems of axes: Show all intersections with the axes clearly. **(6)**

$p(x) = \frac{-4}{x+2} + 1$ and $t(x) = 2$

$y = \frac{-4}{x+2} + 1$

* Shape: $a < 0$ ✓

* Asympt: $x = -2$ $y = 1$ ✓

* x-int: $0 = \frac{-4}{x+2} + 1$ $(y = 0)$

$\frac{4}{x+2} = 1$

$4 = x + 2$

$2 = x$ ✓

$\therefore (2; 0)$

$y = 2$

* y-int: $(x = 0)$

$y = \frac{-4}{0+2} + 1$

$y = \frac{-4}{2} + 1 = -2 + 1 = -1$

$\therefore (0; -1)$ ✓

8.2 Determine the coordinates of p ∩ t. **(1)**

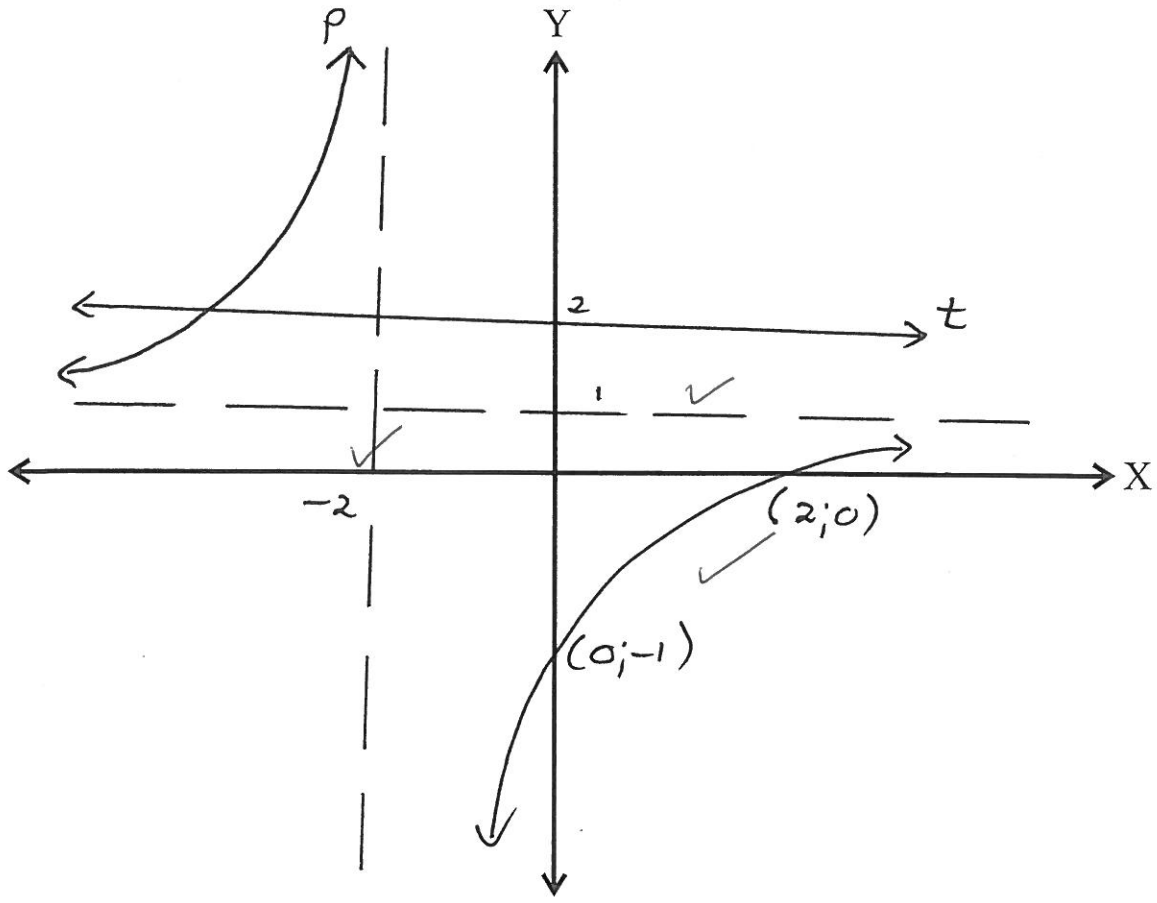
$p(x) = \frac{-4}{x+2} + 1 = 2$

$\frac{-4}{x+2} = 1$

$-4 = x + 2$

$x = -6$

$\therefore (-6; 2)$ ✓


p
Y
2
t
1
X
-2
(2;0)
(0;-1)

Question 9: [8]

9.1 Determine the nature of the roots, without solving the equation:

$4x^2 + 2(x - 1) = 3$ **(4)**

$4x^2 + 2x - 2 = 3$

$4x^2 + 2x - 2 - 3 = 0$

$4x^2 + 2x - 5 = 0$ ✓

$\Delta = b^2 - 4ac$

$= (2)^2 - 4(4)(-5)$

$\Delta = 84$ ✓

$\therefore$ Roots are real; unequal and irrational ✓ ✓

9.2 Determine the value(s) of k if $kx^2 + 2x - 3 = 0$ has non-real roots. **(4)**

$kx^2 + 2x - 3 = 0$

$a = k \quad b = 2 \quad c = -3$

$\Delta = b^2 - 4ac$

$= (2)^2 - 4(k)(-3)$

$\Delta = 4 + 12k$ ✓

Non-real roots $\Rightarrow \Delta < 0$

$\therefore 4 + 12k < 0$ ✓

$12k < -4$

$k < \frac{-4}{12}$

$k < -\frac{1}{3}$ ✓

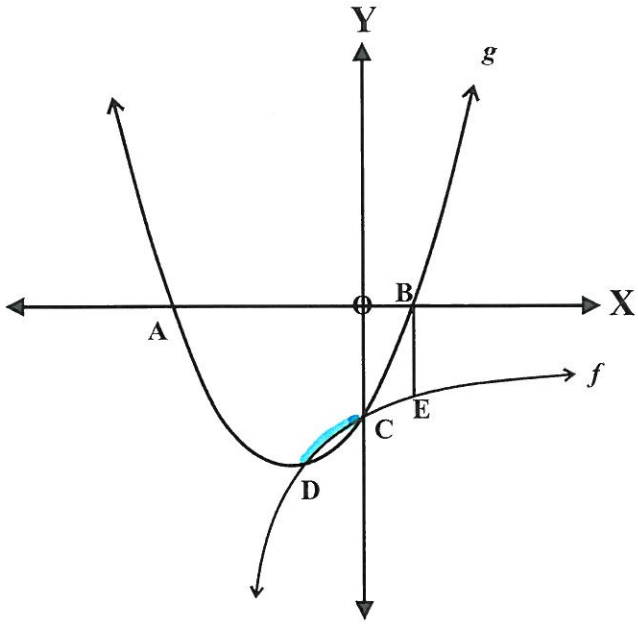
Question 10: [13]

$f(x) = -a^{x+p} - 2$

$g(x) = x^2 + 2x - 3$

D is the turning point of g.

BE ⊥ *x*-axis



10.1 **Determine the length of AB.** **(3)**

10.2 **Determine the coordinates of D.** **(2)**

10.3 **Calculate the values of a and p.** **(3)**

10.4 **Determine the length of BE.** **(3)**

10.5 **Write down the equation of h(x) if h(x) the reflection of f(x) in the y-axis.** **(1)**

10.6 **Determine for which value(s) of x will f(x) > g(x).** **(1)**

10.1. A and B → x-int ($y=0$) of g

$\therefore x^2 + 2x - 3 = 0$ ✓

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

A$(-3;0)$ B$(1;0)$

$\therefore AO = 3$ $OB = 1$

$\therefore AB = 4$ ✓

10.2 Turning point D: $[g(x) = x^2 + 2x - 3]$

$x = \frac{-b}{2a}$ or $x = \frac{x_1 + x_2}{2}$

$= \frac{-(2)}{2(1)}$ $= \frac{-3+1}{2}$

$x = -1$ ✓ $x = -1$

$y = (-1)^2 + 2(-1) - 3$

$= 1 - 2 - 3$

$y = -4$ ✓

$\therefore D(-1; -4)$

10.3 $f(x)$ passes through C and D $(-1; -4)$

C is the y-intercept of $g \rightarrow C(0; -3)$ ✓

$f(x)$:

$\therefore y = -a^{x+p} - 2$ through $C(0; -3)$

$-3 = -a^{0+p} - 2$

$-1 = -a^p$

$1 = a^p$

$a^0 = a^p$

$\therefore p = 0$ ✓

and $y = -a^{x+p} - 2$ through $D(-1; -4)$

$-4 = -a^{-1+0} - 2$

$-2 = -a^{-1}$

$a^{-1} = 2$

$\frac{1}{a} = 2$ $\therefore a = \frac{1}{2}$ ✓

$\therefore f(x) = -\left(\frac{1}{2}\right)^x - 2$

10.4 BE → $x = 1$ B(1;0)

Substitute $x = 1$ ✓ in $f(x)$

$\therefore g(1) = y = -\left(\frac{1}{2}\right)^1 - 2$

$y = -\frac{1}{2} - 2$

$y = -2\frac{1}{2}$

$\therefore BE = 2\frac{1}{2}$ ✓ units

10.5 $h(x) = f(-x)$

$= -\left(\frac{1}{2}\right)^{-x} - 2$

$= -\left(2^{-1}\right)^{-x} - 2$

$\therefore h(x) = -2^x - 2$ ✓

10.6 $-1 < x < 0$ ✓