

- (1) If $\sin 28^\circ = a$ and $\cos 32^\circ = b$, determine the following in terms of a and/or b :

(a) $\cos 28^\circ \rightarrow \cos^2 28^\circ + \sin^2 28^\circ = 1$ (2)

$$\therefore \cos^2 28^\circ = 1 - \sin^2 28^\circ$$

$$\therefore \cos 28^\circ = \sqrt{1 - \sin^2 28^\circ} \quad \checkmark$$

$$\therefore \cos 28^\circ = \sqrt{1 - a^2} \quad \checkmark$$

(b) $\cos 64^\circ = \cos(2 \times 32^\circ) = 2\cos^2 32^\circ - 1 = 2b^2 - 1 \quad \checkmark$ (3)

(c) $\sin 4^\circ = \sin(32^\circ - 28^\circ) \quad \checkmark$ (4)

$$= \sin 32^\circ \times \cos 28^\circ - \cos 32^\circ \times \sin 28^\circ \quad \checkmark$$

$$= \sqrt{1 - \cos^2 32^\circ} \times \sqrt{1 - \sin^2 28^\circ} - \cos 32^\circ \times \sin 28^\circ$$

$$= \sqrt{1 - b^2} \times \sqrt{1 - a^2} - b \times a = \sqrt{1 - b^2} \times \sqrt{1 - a^2} - ab \quad \checkmark \checkmark$$

- (2) Prove without the use of a calculator, that if $\sin 28^\circ = a$ and $\cos 32^\circ = b$, (4)

then: $b\sqrt{1 - a^2} - a\sqrt{1 - b^2} = \frac{1}{2}$

$$\text{LHS} = b\sqrt{1 - a^2} - a\sqrt{1 - b^2}$$

$$\text{RHS} = \frac{1}{2}$$

$$= \cos 32^\circ \sqrt{1 - \sin^2 28^\circ} - \sin 28^\circ \sqrt{1 - \cos^2 32^\circ} \quad \checkmark$$

$$= \cos 32^\circ \sqrt{\cos^2 28^\circ} - \sin 28^\circ \sqrt{\sin^2 32^\circ} \quad \checkmark$$

$$= \cos 32^\circ \cdot \cos 28^\circ - \sin 28^\circ \cdot \sin 32^\circ$$

$$= \cos(32^\circ + 28^\circ) \quad \checkmark$$

$$= \cos(60^\circ) = \frac{1}{2} \quad \checkmark$$

$$\therefore \text{LHS} = \text{RHS}$$

(3) Evaluate each of the following without using a calculator.

Show ALL working.

$$\begin{aligned}
 \text{(a)} \quad & \frac{\sin 130^\circ \cdot \tan 60^\circ}{\cos 540^\circ \cdot \tan 230^\circ \cdot \sin 400^\circ} \quad (7) \\
 &= \frac{\sin(180^\circ - 50^\circ) \cdot \tan 60^\circ}{\cos(540^\circ - 360^\circ) \cdot \tan(180^\circ + 50^\circ) \cdot \sin(360^\circ + 40^\circ)} \\
 &= \frac{\sin 50^\circ \cdot \tan 60^\circ}{\cos 180^\circ \cdot \tan 50^\circ \cdot \sin 40^\circ} \quad \checkmark \checkmark \checkmark \\
 &= \frac{\sin 50^\circ \cdot \sqrt{3}}{(-1) \cdot \frac{\sin 50^\circ}{\cos 50^\circ} \cdot \sin 40^\circ} \quad \checkmark \checkmark \\
 &= \frac{\cancel{\sin 50^\circ} \cdot \sqrt{3}}{(-1) \cdot \sin 40^\circ} \times \frac{\cos 50^\circ}{\cancel{\sin 50^\circ}} \\
 &= \frac{\sqrt{3}}{(-1) \cdot \sin(90^\circ - 50^\circ)} \times \frac{\cos 50^\circ}{1} \\
 &= \frac{\sqrt{3}}{(-1) \cdot \cancel{\cos 50^\circ}} \times \frac{1}{1} \quad \checkmark \\
 &= -\sqrt{3} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & (1 - \sqrt{2} \sin 75^\circ)(\sqrt{2} \sin 75^\circ + 1) \quad (4) \\
 &= 1 - (\sqrt{2} \sin 75^\circ)^2 \quad \checkmark \\
 &= 1 - 2 \sin^2 75^\circ \quad \checkmark \\
 &= \cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ \quad \checkmark \\
 &= -\frac{\sqrt{3}}{2} \quad \checkmark
 \end{aligned}$$

(4) Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$ (7)

$$\begin{aligned}
 & \sin^2 x + \cos^2 x - \sin^2 x - \cos x = 0 \quad \checkmark \\
 & \cos^2 x - \cos x = 0 \\
 & \cos x(\cos x - 1) = 0 \quad \checkmark \\
 & \cos x = 0 \quad \checkmark \quad \text{or} \quad \cos x = 1 \quad \checkmark \\
 & \therefore x = 90^\circ + k \cdot 360^\circ \quad x = 0^\circ + k \cdot 360^\circ \\
 & \therefore x = 90^\circ + k \cdot 360^\circ \quad \checkmark \quad x = k \cdot 360^\circ \quad \checkmark \quad \text{with } k \in \mathbb{Z} \quad \checkmark
 \end{aligned}$$

(5) Consider: $\frac{\cos 2x \cdot \tan x}{\sin^2 x}$

(a) For which values of $x \in [0^\circ; 180^\circ]$, will this expression be undefined? (3)

$$\sin^2 x \neq 0$$

$$\tan x$$

$\therefore$ Undefined if:

$$x = 0^\circ \text{ or } x = 180^\circ \quad \checkmark \checkmark$$

$$x = 90^\circ \quad \checkmark$$

(b) Prove that: $\frac{\cos 2x \cdot \tan x}{\sin^2 x} = \frac{\cos x}{\sin x} - \tan x$ for all other values of x . (5)

$$\begin{aligned} \text{LHS} &= \frac{\cos 2x \cdot \tan x}{\sin^2 x} \\ &= \frac{(\cos^2 x - \sin^2 x) \cdot \frac{\sin x}{\cos x}}{\sin^2 x} \quad \checkmark \checkmark \\ &= \frac{(\cos^2 x - \sin^2 x)}{\sin x \cdot \cos x} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{\cos x}{\sin x} - \tan x \\ &= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \quad \checkmark \\ &= \frac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x} \quad \checkmark \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

(6) The sketch alongside shows one side of the elevation of a house. Some dimensions (in metres) are indicated on the figure.

Calculate, rounded off to ONE decimal place:

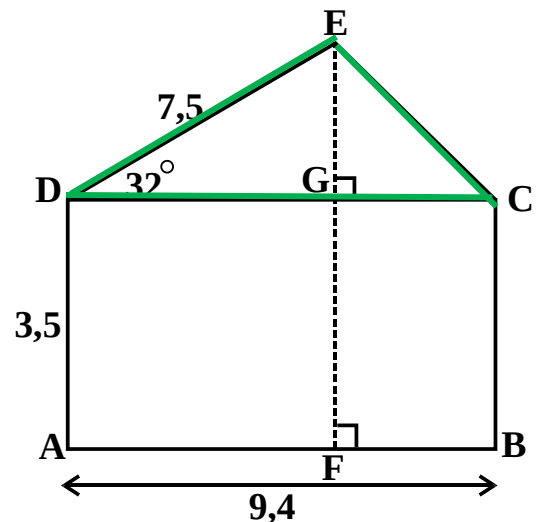
(a) EC In $\triangle CDE$: (3)

$$EC^2 = DE^2 + DC^2 - 2DE \cdot DC \cdot \cos \hat{D} \quad \checkmark$$

$$\therefore EC^2 = (7,5)^2 + (9,4)^2 - 2 \times 7,5 \times 9,4 \times \cos 32^\circ \quad \checkmark$$

$$\therefore EC^2 = 25,035$$

$$\therefore EC \approx 5 \text{ metres} \quad \checkmark$$



(b) $\hat{DCE} \rightarrow$ In $\triangle CDE$: (3)

$$\frac{\sin \hat{DCE}}{DE} = \frac{\sin \hat{D}}{EC} \quad \checkmark$$

$$\therefore \frac{\sin \hat{DCE}}{7,5} = \frac{\sin 32^\circ}{5} \quad \checkmark$$

$$\therefore \sin \hat{DCE} = \frac{7,5 \sin 32^\circ}{5}$$

$$\therefore \sin \hat{DCE} = 0,794 \dots$$

$$\therefore \hat{DCE} \approx 52,6^\circ \quad \checkmark$$

- (c) Area of ΔDEC . $\rightarrow$ In ΔCDE : (2)

$$\begin{aligned}\text{Area } \Delta CDE &= \frac{1}{2} DE \cdot DC \cdot \sin \widehat{DCE} \\ &= \frac{1}{2} \times 7,5 \times 9,4 \sin 32^\circ \checkmark\end{aligned}$$

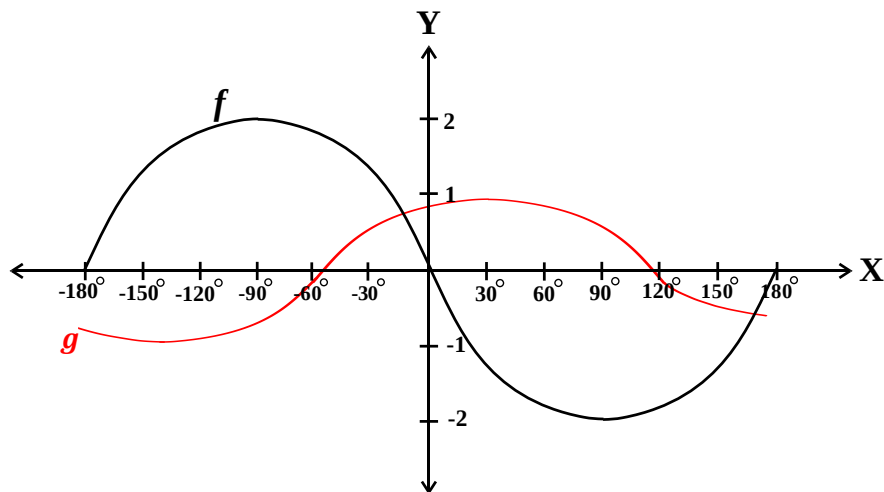
$$\therefore \text{Area } \Delta CDE \approx 18,7 \text{ m}^2 \checkmark$$

- (d) The height of EF. (3)

$$\begin{aligned}\text{Area } \Delta CDE &= \frac{1}{2} DC \times EG \\ \therefore 18,7 &= \frac{1}{2} \times 9,4 \times EG \checkmark \\ \therefore EG &= \frac{18,7 \times 2}{9,4} \\ \therefore EG &\approx 4,0 \checkmark \\ \therefore EF &= EG + GF = 4 + 3,5\end{aligned}$$

$$\therefore EF = 7,5 \text{ metres} \checkmark$$

- (7) The graph of $f(x) = -2 \sin x$ is sketched below.



- (a) Write down the period of f . $360^\circ \checkmark$ (1)

- (b) Write down the amplitude of h if $h(x) = \frac{f(x)}{4}$ (2)

$$\therefore h(x) = \frac{-2 \sin x}{4} = -\frac{1}{2} \sin x \checkmark \rightarrow \text{Amplitude of } h \text{ is } \frac{1}{2} \checkmark$$

- (c) Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $x \in [-180^\circ; 180^\circ]$ (3)

in on the above graph. See in red on the graph

$\checkmark$ Shape $\checkmark$ x -intercepts $\checkmark$ Amplitude

- (d) Use the graph to determine the number of solutions for (1)
 $-2 \sin x = \cos(x - 30^\circ)$, for $x \in [-180^\circ; 180^\circ]$.

Two intercepts ✓

- (e) For which values of x is $g(x) \geq 0$? (2)

$-60^\circ \leq x \leq 120^\circ$ or $x \in [-60^\circ; 120^\circ]$ ✓✓