



**PROVINSIALE EKSAMEN**  
**JUNIE 2023**  
**GRAAD 11**  
**NASIENRIGLYNE**

**WISKUNDE (VRAESTEL 2)**

**10 bladsye**

|               |                          |          |
|---------------|--------------------------|----------|
| NASIENRIGLYNE | WISKUNDE<br>(VRAESTEL 2) | GRAAD 12 |
|---------------|--------------------------|----------|

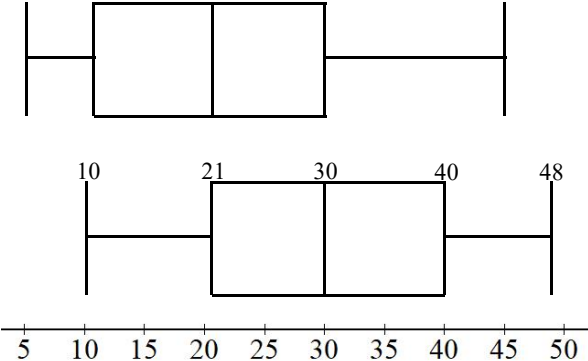
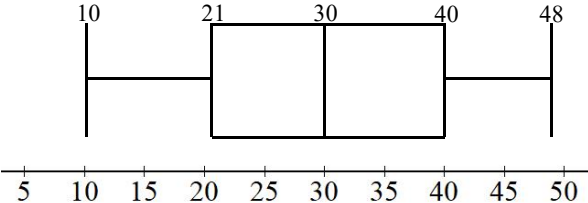
### INSTRUKSIES EN INLIGTING:

- A – AKKURAATHEID
- CA – KONTINUE AKKURAATHEID
- S – BEWERING
- R – REDE
- S & R – BEWERING met REDE

### NOTAS:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en dit nie weer gedoen het nie, merk die doodgetrekte antwoord.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyne toegepas word.
- Dit is onaanvaarbaar om waardes/antwoorde aan te neem om probleme op te los.

VRAAG 1

|     |                                                                                                                                                                                                                                                                                                      |                                                                                                                            |             |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|-------------|
| 1.1 | <p>10; 13; 17; 21; 21; 23; 27; 29; 31; 34; 37; 38; 42; 43; 46; 48</p> <p>Min. = 10 ; <math>Q_1 = 21</math> ; Maks. = 48</p> <p><math>Q_2 = \frac{29+31}{2} = 30</math> ; <math>Q_3 = \frac{38+42}{2} = 40</math></p>                                                                                 | <p>✓ Korrekte Min.<br/><math>Q_1</math> en Maks.</p> <p>✓ Korrekte <math>Q_2</math></p> <p>✓ Korrekte <math>Q_3</math></p> | (3)         |
| 1.2 | <p>Graad 11A</p>  <p>Graad 11B</p>  <p>5 10 15 20 25 30 35 40 45 50</p>                                                          | <p>✓ Korrekte Min./<br/>onderste punt</p> <p>✓ Korrekte Maks./<br/>boonste punt</p> <p>✓ Korrekte houer</p>                | (3)         |
| 1.3 | <p><math>\bar{x} = \frac{480}{16}</math></p> <p><math>\bar{x} = 30</math></p>                                                                                                                                                                                                                        | <p>✓ Breuk</p> <p>✓ Antwoord</p> <p><b>NB:</b> Antwoord alleen<br/>volpunte</p>                                            | (2)         |
| 1.4 | $\sigma = 11,46$                                                                                                                                                                                                                                                                                     | ✓✓ Antwoord                                                                                                                | (2)         |
| 1.5 | <p>1σ interval: (30 – 11,46 ; 30 + 11,46)<br/>(18,54 ; 41,46)</p> <p>4 leerders het meer as 41,46 punte behaal.</p>                                                                                                                                                                                  | <p>✓ Interval</p> <p>✓ Antwoord</p>                                                                                        | (2)         |
| 1.6 | <p>Daar is geen verskil in die IKO nie. IKO = 19 vir albei klasse.<br/>Alhoewel, die middel 50% van Graad 11B tussen 21 en 40 was in<br/>vergelyking met Graad 11A waar 50% tussen 11 en 30 was. Die<br/>houer vir 11B het na regs geskuif. Dus het Graad 11B oor die<br/>algemeen beter gevaar.</p> | <p>✓ Opmerking</p> <p>✓ IKO waarde</p>                                                                                     | (2)         |
| 1.7 | 75%                                                                                                                                                                                                                                                                                                  | ✓ Antwoord                                                                                                                 | (1)         |
|     |                                                                                                                                                                                                                                                                                                      |                                                                                                                            | <b>[15]</b> |

## VRAAG 2

|     |                                                                                                                                                                                                                 |                                                                                                     |             |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|-------------|
| 2.1 | <p>Vergelyking van EG:</p> $3x - y + 6 = 0$ $y = 3x + 6$ <p>dan <math>m_{EG} = 3</math></p> $\tan \beta = 3$ $\beta = 71,57^\circ$                                                                              | <p>✓ Gradiënt</p> <p>✓ Antwoord</p>                                                                 | (2)         |
| 2.2 | $x + 3(0) = 8$ $x = 8$ $\therefore (8; 0)$                                                                                                                                                                      | <p>✓ <math>x = 0</math></p> <p>✓ Koördinaat vorm</p>                                                | (2)         |
| 2.3 | <p><math>M_{DF} = M_{GE}</math> hoeklyne halveer mekaar</p> $K\left(\frac{-10+8}{2}; \frac{6+0}{2}\right)$ $K(-1; 3)$ $-1 = \frac{x+1}{2} \quad 3 = \frac{y+9}{2}$ $x = -3 \quad y = -3$ $\therefore E(-3; -3)$ | <p>✓ <math>M_{DE} = M_{GE}</math></p> <p>✓ Koördinate van K</p> <p>✓ x-waarde</p> <p>✓ y-waarde</p> | (4)         |
| 2.4 | $m_{DF} \times m_{GE}$ $= -\frac{1}{3} \times 3$ $= -1$ $\therefore DF \perp GE$ <p>DGFE is 'n ruit. (hoeklyne halveer mekaar loodreg)</p>                                                                      | <p>✓ <math>m_{DF}</math></p> <p>✓ <math>DF \perp GE</math></p> <p>✓ Rede</p>                        | (3)         |
|     |                                                                                                                                                                                                                 |                                                                                                     | <b>[11]</b> |

### VRAAG 3

|     |                                                                                                                                         |                                                  |             |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|-------------|
| 3.1 | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$<br>$AC = \sqrt{(1 - (-2))^2 + (5 - (-1))^2}$<br>$AC = 3\sqrt{5}$                             | ✓ Substitusie<br><br>✓ Antwoord                  | (2)         |
| 3.2 | $m_{BC} = \frac{5 - 4}{1 - (-3)}$<br>$m_{BC} = \frac{1}{4}$                                                                             | ✓ Substitusie<br>✓ Antwoord                      | (2)         |
| 3.3 | $m_{BC} = m_{BD} = m_{CD}$<br>$\frac{1}{4} = \frac{y - 5}{5 - 1} \quad \text{of} \quad \frac{1}{4} = \frac{y - 4}{5 - (-3)}$<br>$y = 6$ | ✓ Gelyke gradiënte<br>✓ Substitusie<br>✓ $y = 6$ | (3)         |
| 3.4 | $m_{AH} = -4 \quad (AH \perp BC)$<br>$y - (-1) = -4(x - (-2))$<br>$y = -4x - 9$                                                         | ✓ $m_{AH}$<br>✓ Vervang A<br>✓ Vergelyking       | (3)         |
|     |                                                                                                                                         |                                                  | <b>[10]</b> |

### VRAAG 4

|     |       |                                    |                                                                                                     |     |
|-----|-------|------------------------------------|-----------------------------------------------------------------------------------------------------|-----|
| 4.1 | 4.1.1 | $x^2 + (-24)^2 = 25^2$<br>$x = -7$ | ✓ Vervang in Pythagoras<br>✓ Antwoord                                                               | (2) |
|     | 4.1.2 | (a)                                | $\sin \beta$<br>$= -\frac{24}{25}$                                                                  | (1) |
|     |       | (b)                                | $\cos(180^\circ - \beta)$<br>$= -\cos \beta$<br>$= -\left(\frac{-7}{25}\right)$<br>$= \frac{7}{25}$ | (2) |

|  |       |     |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                |     |
|--|-------|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
|  |       | (c) | $\tan^2(-\beta)$<br>$= \tan^2 \beta$<br>$= \left(\frac{-24}{-7}\right)^2$<br>$= \frac{576}{49}$                                                                                                                                                                                                                                                   | ✓ Reduseer<br>✓ Vereenvoudigde antwoord                                                                                                                                                        | (2) |
|  |       |     |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                |     |
|  | 4.1.3 | (a) | $\sin \beta = \frac{-24}{25}$<br>$\frac{y}{15} = \frac{-24}{25}$<br>$y = \frac{-72}{5}$<br>$\cos \beta = \frac{-7}{25}$<br>$\frac{x}{15} = \frac{-7}{25}$<br>$x = \frac{-21}{5}$                                                                                                                                                                  | ✓ $\sin \beta = \frac{y}{15}$<br>✓ Stel sin verhoudings gelyk<br>✓ $\cos \beta = \frac{x}{15}$<br>✓ Stel cos verhoudings gelyk                                                                 | (4) |
|  |       |     |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                |     |
|  |       | (b) | $\text{Opp}\Delta\text{ROT} = \frac{1}{2}(12)\left(\frac{72}{5}\right)$<br>$\text{Opp}\Delta\text{ROT} = 86,4 \text{ units}^2$<br><b>ALTERNATIEF</b><br>$OT = 15 \quad OF = 12$<br>$\beta = 253,7397\dots^\circ$<br>$\text{Opp}\Delta\text{ROT} = \frac{1}{2}(15)(12) \sin (106,26^\circ)$<br>$\text{Opp}\Delta\text{ROT} = 86,4 \text{ units}^2$ | ✓ Formule vir oppervlak van $\Delta$<br>✓ Basis = 12<br>✓ Hoogte = $\frac{72}{5}$<br>✓ Oppervlak<br><br>✓ OT lengte<br>✓ $106,26^\circ$<br>✓ Vervang in sin oppervlak formule<br>✓ Oppervlakte | (4) |

|     |                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                            |      |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 4.2 | $\frac{\tan 225^\circ + \sin (180^\circ - \theta) \cos (90^\circ + \theta)}{\cos (90^\circ - \theta) \sin (-\theta - 540^\circ)}$ $= \frac{1 + \sin \theta \cdot -\sin \theta}{\sin \theta \cdot \sin \theta}$ $= \frac{1 - \sin^2 \theta}{\sin^2 \theta}$ $= \frac{\cos^2 \theta}{\sin^2 \theta}$ $= \tan^2 \theta$ | <ul style="list-style-type: none"> <li>✓ <math>\tan 225^\circ = 1</math></li> <li>✓ <math>\sin (180^\circ - \theta) = \sin \theta</math></li> <li>✓ <math>\cos (90^\circ + \theta) = -\sin \theta</math></li> <li>✓ <math>\sin (-\theta - 540^\circ) = \sin \theta</math></li> <li>✓ <math>1 - \sin^2 \theta = \cos^2 \theta</math></li> <li>✓ <math>\frac{\cos^2 \theta}{\sin^2 \theta} = \tan^2 \theta</math></li> </ul> | (6)  |
|     |                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                            | [21] |

### VRAAG 5

|     |                                                                          |                                                                                                                    |     |
|-----|--------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|-----|
|     |                                                                          |                                                                                                                    |     |
| 5.1 | $A = 2$                                                                  | ✓ Antwoord                                                                                                         | (1) |
| 5.2 | Periode = $720^\circ$                                                    | ✓ Antwoord                                                                                                         | (1) |
| 5.3 | $a = -1$<br>$p = -60^\circ$                                              | <ul style="list-style-type: none"> <li>✓ Waarde van <math>a</math></li> <li>✓ Waarde van <math>p</math></li> </ul> | (2) |
| 5.4 | $-120^\circ \leq x \leq 60^\circ$ of $x = -240^\circ$ of $x = 240^\circ$ | <ul style="list-style-type: none"> <li>✓ Interval</li> <li>✓ <math>-240^\circ; 240^\circ</math></li> </ul>         | (2) |
| 5.5 | $p = 30^\circ$                                                           | ✓✓ Waarde van $p$                                                                                                  | (2) |
|     |                                                                          |                                                                                                                    | [8] |

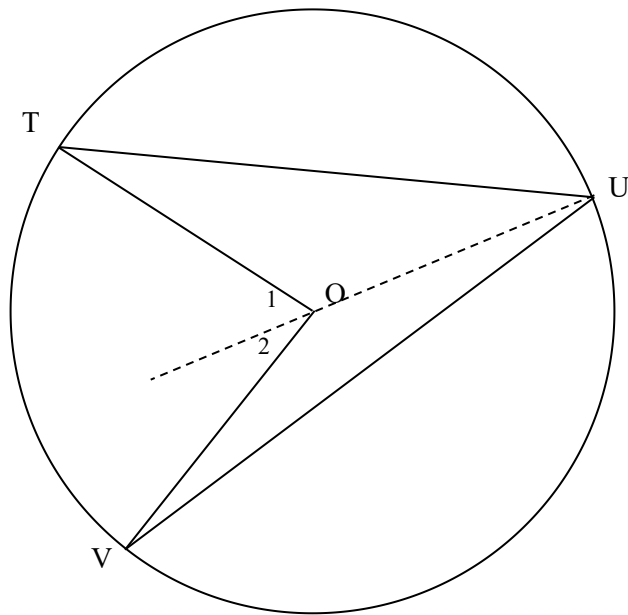
## VRAAG 6

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                       |     |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 6.1 | $\tan\left(\frac{\theta}{2}\right) = \frac{r}{R}$<br>$R \cdot \tan\left(\frac{\theta}{2}\right) = r$                                                                                                                                                                                                                                                                                                                                                                                                                                                              | ✓ Substitusie<br>✓ Manipulasie                                                                                                                        | (2) |
| 6.2 | $V_{\text{hemisfeer}} = \frac{1}{2} \cdot \frac{4}{3} \pi R^3$<br>$V_{\text{hemisfeer}} = \frac{2}{3} \pi R^3$<br>$V_{\text{keël}} = \frac{1}{3} \pi r^2 R$<br>$V_{\text{keël}} = \frac{1}{3} \pi \left( R \cdot \tan\left(\frac{\theta}{2}\right) \right)^2 R$<br>$V_{\text{keël}} = \frac{2}{3} \pi R^3 \tan^2\left(\frac{\theta}{2}\right)$<br>$V_{\text{oorblywende}} = \frac{2}{3} \pi R^3 - \frac{1}{3} \pi R^3 \tan^2\left(\frac{\theta}{2}\right)$<br>$V_{\text{oorblywende}} = \frac{\pi R^3}{3} \left( 2 - \tan^2\left(\frac{\theta}{2}\right) \right)$ | ✓ $\frac{1}{2}$ Volume Sfeer<br>✓ Vervang $r$ uit 6.1<br>✓ Trek volumes af (Metode)<br>✓ Faktore                                                      | (4) |
| 6.3 | $V_{\text{keël}} = \frac{1}{3} \pi R^3 \tan^2\left(\frac{\theta}{2}\right)$<br>$V_{\text{keël}} = \frac{1}{3} \pi (11)^3 \tan^2\left(\frac{36^\circ}{2}\right)$<br>$V_{\text{keël}} = 147,15 \text{ cm}^3$                                                                                                                                                                                                                                                                                                                                                        | ✓ $V_{\text{keël}} = \frac{1}{3} \pi R^3 \tan^2\left(\frac{\theta}{2}\right)$<br>✓ Vervang<br>$R = 11$ en $\theta = 36^\circ$<br>✓ Volume van kerswas | (3) |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                       | [9] |

## VRAAG 7

|     |       |                        |                                |            |     |
|-----|-------|------------------------|--------------------------------|------------|-----|
| 7.1 | 7.1.1 | $\hat{C}_2 = 40^\circ$ | raaklyn koord stelling         | ✓ S    ✓ R | (2) |
|     | 7.1.2 | $\hat{K}_1 = 90^\circ$ | $\angle$ in 'n halwe sirkel    | ✓ S    ✓ R | (2) |
|     | 7.1.3 | $\hat{P}_1 = 25^\circ$ | buite $\angle$ van 'n $\Delta$ | ✓ S    ✓ R | (2) |
| 7.2 |       | $\hat{T}_1 = 65^\circ$ | buite $\angle$ van $\Delta$    | ✓ R        |     |
|     |       | TC = QC                | sye teen gelyke $\angle e$     | ✓ R        | (2) |
|     |       |                        |                                |            | [8] |

VRAAG 8



8.1 *Konstrueer  $UO$  en verleng.*

In  $\triangle TOU$

$$\hat{O}_1 = \hat{T} + \hat{U}_1 \quad \text{buite } \angle \text{ van } \triangle$$

$$\hat{T} = \hat{U}_1 \quad \angle \text{e teen gelyke radii}$$

$$\therefore \hat{O}_1 = 2\hat{U}_1$$

Net so in  $\triangle TOU$

$$\therefore \hat{O}_2 = 2\hat{U}_2$$

$$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{U}_1 + 2\hat{U}_2$$

$$\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{U}_1 + \hat{U}_2)$$

$$\hat{TOV} = 2\hat{TUV}$$

✓ Konstruksie

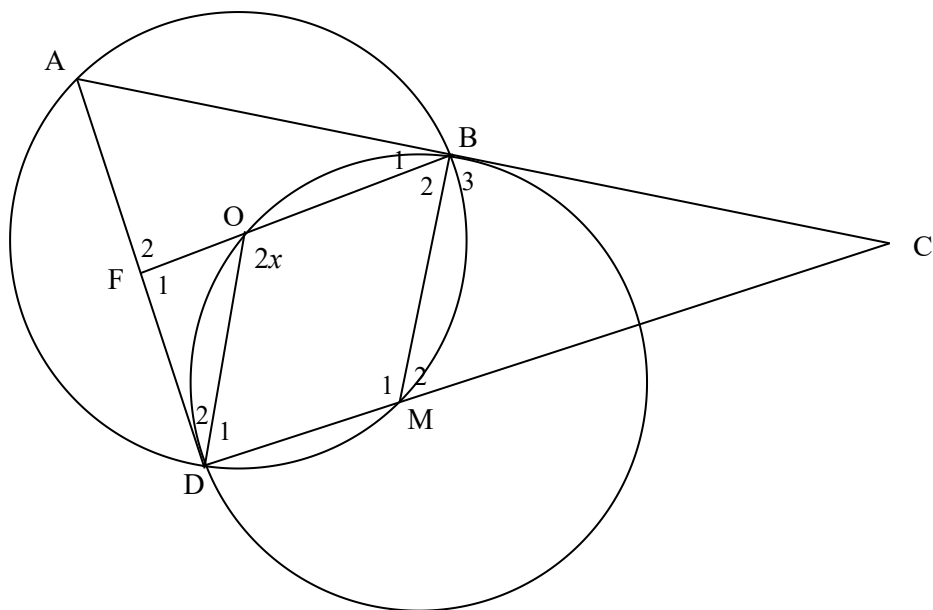
✓ SR

✓ SR

✓ S

✓ Gevolgtrekking

(5)



|                |       |                                   |                                               |                  |             |
|----------------|-------|-----------------------------------|-----------------------------------------------|------------------|-------------|
| 8.2            | 8.2.1 | Twee pare aangrensende sye gelyk. |                                               | ✓ R              | (1)         |
|                | 8.2.2 | (a)                               | midpts $\angle = 2 \times$ omtreks $\angle$   | ✓ R              | (1)         |
|                |       | (b)                               | buite $\angle$ van koordevierhoek             | ✓ R              | (1)         |
|                |       | (c)                               | ooreenkomstige $\angle e$ , $OD \parallel MB$ | ✓ R              | (1)         |
|                |       | (d)                               | teenoorst $\angle e$ , ruit                   | ✓ R              |             |
|                |       | <b>ALTERNATIEF</b>                |                                               |                  |             |
|                |       |                                   | verw $\angle e$ , $OB \parallel DM$           |                  | (1)         |
|                | 8.2.3 | $\hat{D}_2 + x = 90^\circ$        | raaklyn $\perp$ rad                           | ✓ R              |             |
|                |       | $\hat{D}_2 = 90^\circ - x$        |                                               |                  |             |
|                |       | $\hat{B}_3 = 90^\circ$            | raaklyn $\perp$ rad                           | ✓ S              |             |
|                |       | $\hat{C} = 90^\circ - x$          | binne $\angle e$ van $\Delta$                 | ✓ SR             |             |
|                |       | $\therefore \hat{D}_2 = \hat{C}$  | albei $90^\circ - x$                          | ✓ Gevolgtrekking | (4)         |
|                | 8.2.4 | $\hat{F}_2 = 90^\circ$            | ooreenk $\angle e$ , $FB \parallel DM$        | ✓ SR             |             |
|                |       | $AF = FD$                         | lyn van midpt $\perp$ koord                   | ✓ S              |             |
|                |       | $AB = BC$                         | lyn deur midpt $\parallel$ 2 <sup>de</sup> sy | ✓ R              |             |
|                |       |                                   |                                               | ✓ R              | (4)         |
|                |       |                                   |                                               |                  | <b>[18]</b> |
|                |       |                                   |                                               |                  |             |
| <b>TOTAAL:</b> |       |                                   |                                               |                  | <b>100</b>  |