LIMPOPO

PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

LIMPOPO PROVINCE

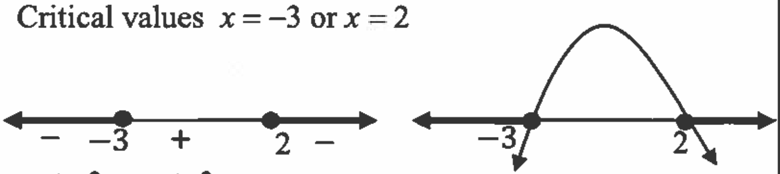
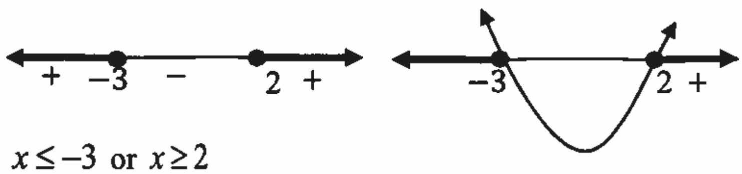
GRADE 12

MATHEMATICS P1
MEMORANDUM
SEPTEMBER 2021

MARKS : 150

This memorandum consists of 17 pages.

QUESTION 1

1.1	1.1.1	$x^2+4x-45=0$ $(x+9)(x-5)=0$ $x=-9$ or $x=5$	✓both factors ✓ $x=-9$ ✓ $x=5$	(3)
	1.1.2	$7x^2-14x+3=0$ $x=\frac{-(-14)\pm\sqrt{(-14)^2-4(7)(3)}}{2(7)}$ $x=\frac{14\pm\sqrt{112}}{14}$ $x=1,76$ or $x=0,24$	✓substitution into the correct formula ✓ $x=1,76$ ✓ $x=0,24$	(3)
	1.1.3	$(x+3)(2-x)\le 0$ Critical values $x=-3$ or $x=2$  $x\le -3$ or $x\ge 2$ OR $(x+3)(x-2)\ge 0$ Critical values $x=-3$ or $x=2$  $x\le -3$ or $x\ge 2$	✓critical values ✓✓answer OR ✓critical values ✓✓answer	(3)
	1.1.4	$\sqrt{x+34}-x=4$ $\sqrt{x+34}=x+4$ $\left(\sqrt{x+34}\right)^2=(x+4)^2$ $x+34=x^2+8x+16$ $x^2+7x-18=0$ $(x+9)(x-2)=0$ $x\ne -9$ or $x=2$	✓isolating the square root ✓ $x+34=x^2+8x+16$ ✓standard form ✓ $x\ne -9$ ✓ $x=2$	(5)

	1.1.5	$\frac{3^y \times 9^{x-y}}{27^{x-y}}=1$ $\frac{3^y \times 3^{2x-2y}}{3^{3x-3y}}=1$ $3^{2y-x}=3^0$ $2y-x=0$ $x=2y$	✓ 3^{2x-2y} ✓ 3^{3x-3y} ✓ $3^{2y-x}=3^0$ ✓ $x=2y$	(4)
1.2		$\left(\sqrt{\sqrt{4}-\sqrt{3}}\right)\left(\sqrt{\sqrt{4}+\sqrt{3}}\right)$ $=\sqrt{\left(\sqrt{4}\right)^2-\left(\sqrt{3}\right)^2}$ $=\sqrt{4-3}$ $=1$	✓ $\sqrt{\left(\sqrt{4}\right)^2-\left(\sqrt{3}\right)^2}$ ✓ $\sqrt{4-3}=1$	(2)
1.3		$y=2-3x$ ....(1) $x^2+y=xy+x$ ....(2) substitute (1) into (2) $x^2+(2-3x)=x(2-3x)+x$ $x^2-3x+2=2x-3x^2+x$ $4x^2-6x+2=0$ $2(2x-1)(x-1)=0$ $x=\frac{1}{2}$ or $x=1$ $y=\frac{1}{2}$ or $y=-1$	✓ $y=2-3x$ ✓substitution ✓standard form ✓both values of x ✓both values of y	

		OR $x=\frac{2-y}{3}$ ....(1) $x^2+y=xy+x$............................(2) substitute (1) into (2) $\left(\frac{2-y}{3}\right)^2+y=\left(\frac{2-y}{3}\right)y+\frac{2-y}{3}$ $\left(\frac{4-4y+y^2}{9}\right)+y=\frac{2y-y^2}{3}+\frac{2-y}{3}$ $y^2+5y+4=6+3y-3y^2$ $2y^2+y-1=0$ $y=\frac{1}{2}$ or $y=-1$ $x=\frac{1}{2}$ or $x=1$	OR ✓$x=\frac{2-y}{3}$ ✓substitution ✓standard form ✓both values of y ✓both values of x	(5)
				[25]

QUESTION 2

2.1	2.1.1	$2p-2-(2p+1)=4p-3-(2p-2)$ $-3=2p-1$ $p=-1$	✓equating ✓answer	(2)
	2.1.2	$-1;\ -4;\ -7$	✓✓answers	(2)
	2.1.3	$a=-1$ and $d=-3$ $T_n=a+(n-1)d$ $T_{50}=-1+49(-3)$ $=-148$	✓substitution ✓answer	(2)

2.2	2.2.1	3 11 21 33 47 8 10 12 14 2 2 2 $2a=2$ $a=2$	✓2nd difference ✓$a=1$	(2)
	2.2.2	$3a+b=8$ $a+b+c=3$ $3(1)+b=8$ $1+5+c=3$ $b=5$ $c=-3$ $T_n=n^2+5n-3$ $\therefore n^2+5n-3=497$ $n^2+5n-500=0$ $(n+25)(n-20)=0$ $n=-25$ or $n=20$ $\therefore n=20$ $T_{20}=497$	✓$b=5$ ✓$c=-3$ ✓setting up an equation ✓standard form ✓factors ✓choosing $n=20$	(6)
				[15]

QUESTION 3

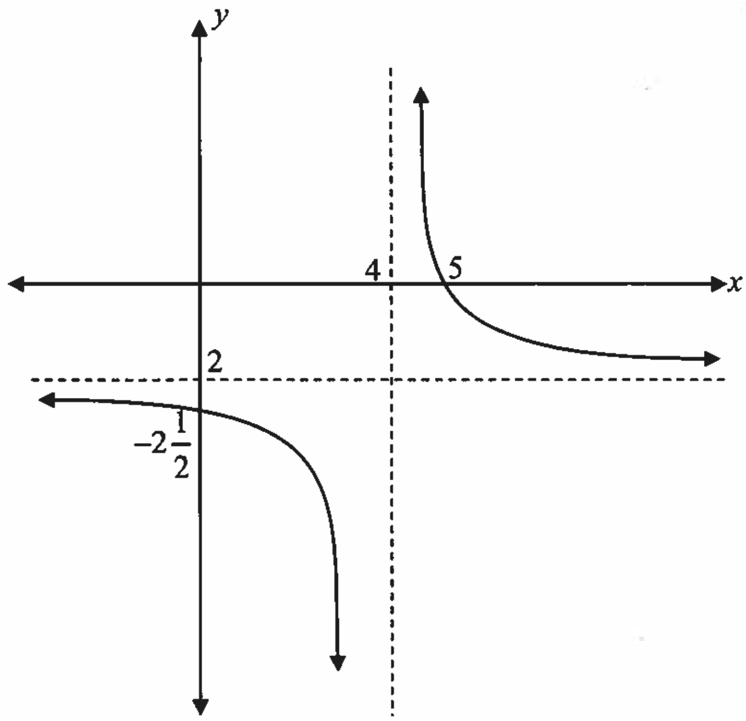
3.1		$ar=24$ $\sum_{n}^{\infty} T_n=\frac{a}{1-r}=100$ $a=100(1-r)$ $r=\frac{24}{a}$ $a=100\left(1-\frac{24}{a}\right)$ $a=100-\frac{2400}{a}$ $a^2-100a+2400=0$ $(a-40)(a-60)=0$ $a=40$ or $a=60$	✓both $\frac{a}{1-r}=100$ and and $ar=24$ ✓$a=100(1-r)$ ✓$r=\frac{24}{a}$ ✓substitution ✓standard form ✓both values of a	

		OR $ar = 24$ $\sum_{n}^{\infty} T_n = \frac{a}{1-r} = 100$ $\therefore a = 100(1-r)$ $\Rightarrow 100(1-r)r = 24$ $100r - 100r^2 = 24$ $100r^2 - 100r + 24 = 0$ $25r^2 - 25r + 6 = 0$ $(5r-2)(5r-3) = 0$ $r = \frac{2}{5}$ or $r = \frac{3}{5}$ $\therefore a = 60$ or $a = 40$	OR ✓both $\frac{a}{1-r} = 100$ and and $ar = 24$ ✓ $a = 100(1-r)$ ✓substitution ✓standard form ✓both values of r ✓both values of a	(6)
3.2		$\sum_{k=1}^{m} 2.2^k < 131068$ 4+8+16+…to m terms $S_n = \frac{a(r^n - 1)}{r-1} = \frac{4(2^m - 1)}{2-1}$ $\frac{4(2^m - 1)}{2-1} < 131068$ $2^m - 1 < 32767$ $2^m < 32768$ $2^m < 2^{15}$ $m < 15$ $m = 14$	✓4+8+16+…to m terms ✓substitution into the correct formula ✓ $2^m < 32768$ ✓ $2^m < 2^{15}$ ✓ $m = 14$	

		OR 4+8+16+…to m terms $\frac{4(2^m-1)}{2-1}<131068$ $2^m-1<32767$ $2^m<32768$ $\log 2^m<\log 32768$ $m<\frac{\log 32768}{\log 2}$ $m<15$ $m=14$	OR ✓4+8+16+…to m terms ✓substitution into the correct formula ✓ $2^m<32768$ ✓correct use of logs ✓ $m=14$	(5)
				[11]

QUESTION 4

4.1		$x=4$ $y=-2$	✓ $x=4$ ✓ $y=-2$	(2)
4.2		x-intercept: $y=0$ $\frac{2}{x-4}-2=0$ $2=2(x-4)$ $x=5$	✓ $y=0$ ✓answer	(2)

4.3			✓x-intercept ✓y-intercept ✓both asymptotes ✓shape	(4)
4.4		$y=-x+c$ $-2=-4+c$ $c=2$ $y=-x+2$	✓use of point $(4\ ;\ -2)$ ✓value of c	(2)
				[10]

QUESTION 5

5.1		$y-y_1=m(x-x_1)$ $y=\frac{-8}{3+1}(x+1)$ $=-2(x+1)$ $=-2x-2$	✓substitution of points T and R into the correct formula ✓gradient ✓answer	

		OR The gradient $m=\frac{-8}{4}=-2$ $y=mx+c$ $-8=-2(3)+c$ $c=-2$ $y=-2x-2$	OR ✓gradient ✓substitution of point R ✓answer	(3)
5.2		$y=a(x-x_1)(x-x_2)$ $=a(x+1)(x-5)$ $-8=a(3+1)(3-5)$ $a=1$ $y=(x+1)(x-5)$ $y=x^2-4x-5$	✓substitution of points T and W into the correct formula ✓use of point R. ✓value of a ✓value of b ✓value of c	(5)
5.3	5.3.1	$x=-\frac{b}{2a}$ $x=-\frac{-4}{2(1)}$ $x=2$ $f(2)=-9$ $\text{V}(2\ ;\ -9)$ OR $x=\frac{x_1+x_2}{2}$ $x=\frac{-1+5}{2}$ $x=2$ $f(2)=-9$ $\text{V}(2\ ;\ -9)$	✓substitution of c and b ✓value of x ✓ $f(2)=-9$ OR ✓ $x=\frac{-1+5}{2}$ ✓value of x ✓ $f(2)=-9$	

		OR $f'(x)=2x-4$ $2x-4=0$ $x=2$ $f(2)=-9$ $V(2\ ;\ -9)$	OR ✓ $f'(x)=0$ ✓value of x ✓ $f(2)=-9$	(3)
	5.3.2	$\therefore$ Range: $y\geq -9$ OR $y\in[-9;\infty)$	✓✓answer	(2)
5.4		$f(x)=x^2-4x-5$ and turning point $(2\ ;\ -9)$ $f(x)=a(x+p)^2+q$ $f(x)=(x-2)^2-9$ $k(x)=(x-4-2)^2-9+5$ $k(x)=(x-6)^2-4$	✓ $f(x)=(x-2)^2-9$ ✓ $x-6$ ✓ -4	(3)
5.5		$-1\leq x\leq 2$	✓✓answer	(2)
				[18]

QUESTION 6

6.1		R(0;1)	✓answer	(1)
6.2		$y=b^x$ $64=b^{-3}$ $b^3=\frac{1}{4^3}$ $b=\frac{1}{4}$	✓substitution of point P(–3 ; 64) ✓ $b^3=\frac{1}{4^3}$ ✓value of b.	(3)
6.3		$y=\left(\frac{1}{4}\right)^x$ $x=\left(\frac{1}{4}\right)^y$ $p^{-1}(x)=\log_{\frac{1}{4}} x$	✓swapping x and y ✓answer	(2)

6.4		$0<\log_{\frac{1}{4}} x<1$ $\left(\frac{1}{4}\right)^0 > \left(\frac{1}{4}\right)^{\log_{\frac{1}{4}} x} > \frac{1}{4}$ $\therefore \frac{1}{4}<x<1$ OR $\Rightarrow \log_{\frac{1}{4}} x>0$ and $\log_{\frac{1}{4}} x<1$ $\Rightarrow x<\left(\frac{1}{4}\right)^0$ and $x>\left(\frac{1}{4}\right)^1$ $\therefore \frac{1}{4}<x<1$	✓exponential form ✓answer OR ✓method ✓answer	(2)
				[8]

QUESTION 7

7.1		The value of the car $A=P(1-i)^n$ $A=P(1-0{,}15)^5$ $=155\ 296, 85$	✓substitution into correct formula ✓value of i ✓answer	(3)
7.2	7.2.1	$P=\dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $500000=\dfrac{9500\left[1-\left(1+\frac{0{,}115}{12}\right)^{-n}\right]}{\frac{0{,}115}{12}}$ $-n=\dfrac{\log\left[1-\dfrac{500000\left(\frac{0{,}115}{12}\right)}{9500}\right]}{\log\left(1+\frac{0{,}115}{12}\right)}$ $n=73{,}59819249$ months $=74$ months	✓value of i ✓substitution into the correct formula ✓correct use of logs ✓74 months	(4)

	7.2.2	The balance at the end of 73 months: $P=\dfrac{9500\left[1-\left(1+\frac{0,115}{12}\right)^{-0,59819249}\right]}{\frac{0,115}{12}}$ $=R5\ 639,667634$ The final payment will be: $A=P(1+i)^n$ $=5639,667634\left(1+\frac{0,115}{12}\right)^1$ $=R5\ 693,71$ OR $\text{Balance}=A-F_v$ $=500000\left(1+\frac{0,115}{12}\right)^{73}-\dfrac{9500\left(\left(1+\frac{0,115}{12}\right)^{73}-1\right)}{\frac{0,115}{12}}$ R5 639,667634 The final payment will be: $A=P(1+i)^n$ $=5639,667634\left(1+\frac{0,115}{12}\right)^1$ $=R5\ 693,71$	✓value of n ✓substitution into correct formula ✓R5 639,67 ✓$n=1$ ✓R5 693,71 OR ✓value of n ✓substitution into correct formulae ✓R5 639,67 ✓$n=1$ ✓R5 693,71	

		OR $$500000=\frac{9500\left[1-\left(1+\frac{0,115}{12}\right)^{-73}\right]}{\frac{0,115}{12}}+x\left(1+\frac{0,115}{12}\right)^{-74}$$ $$x=\frac{500000-\frac{9500\left[1-\left(1+\frac{0,115}{12}\right)^{-73}\right]}{\frac{0,115}{12}}}{\left(1+\frac{0,115}{12}\right)^{-74}}$$ $x=$R5 693,71	OR ✓$n=-73$ ✓$n=-74$ ✓$9500\left[1-\left(1+\frac{0,115}{12}\right)^{-73}\right]$ ✓$x\left(1+\frac{0,115}{12}\right)^{-74}$ ✓answer	(5)
	7.2.3	Interest = Total repayments – (Loan amount – Last repayment) = R9 500 × 73 – (R500000 – R5 639,67) = R199 139,67	✓method ✓answer	(2)
				[14]

QUESTION 8

8.1		$f(x)=-2x^2+1$ $f(x+h)=-2(x+h)^2+1$ $=-2x^2-4xh-2h^2+1$ $f'(x)=\lim_{h\to0}\frac{f(x+h)+f(x)}{h}$ $=\lim_{h\to0}\frac{-2x^2-4xh-2h^2+1-(-2x^2+1)}{h}$ $=\lim_{h\to0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to0}\frac{h(-4x-2h)}{h}$ $=\lim_{h\to0}(-4x-2h)$ $=-4x$	✓expansion ✓correct substitution ✓simplification ✓common factor ✓answer	

		OR $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)+f(x)}{h}$ $=\lim_{h\to 0}\dfrac{-2(x+h)^2+1-(-2x^2+1)}{h}$ $=\lim_{h\to 0}\dfrac{-2x^2-4xh-2h^2+1+2x^2-1}{h}$ $=\lim_{h\to 0}\dfrac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\dfrac{h(-4x-2h)}{h}$ $=\lim_{h\to 0}(-4x-2h)$ $=-4x$	OR ✓correct substitution ✓expansion ✓simplification ✓common factor ✓answer	(5)
8.2	8.2.1	$y=2-x-x^3$ $\dfrac{dy}{dx}=-1-3x^2$	✓ -1 ✓ $-3x^2$	(2)
	8.2.	$D_x\left(-\dfrac{2x}{\sqrt{x}}-\dfrac{1}{x}\right)$ $=D_x\left(-2x^{\frac{1}{2}}-x^{-1}\right)$ $=-x^{-\frac{1}{2}}+x^{-2}$	✓ $-2x^{\frac{1}{2}}$ ✓ $-x^{-1}$ ✓ $-x^{-\frac{1}{2}}$ ✓ x^{-2}	(4)
	8.2.2	$f(x)=\dfrac{2x^2-16x+14}{2x-2}$ $=\dfrac{2(x-1)(x-7)}{2(x-1)}$ $=x-7$ $f'(x)=1$	✓factors ✓simplification ✓answer	(3)
				[14]

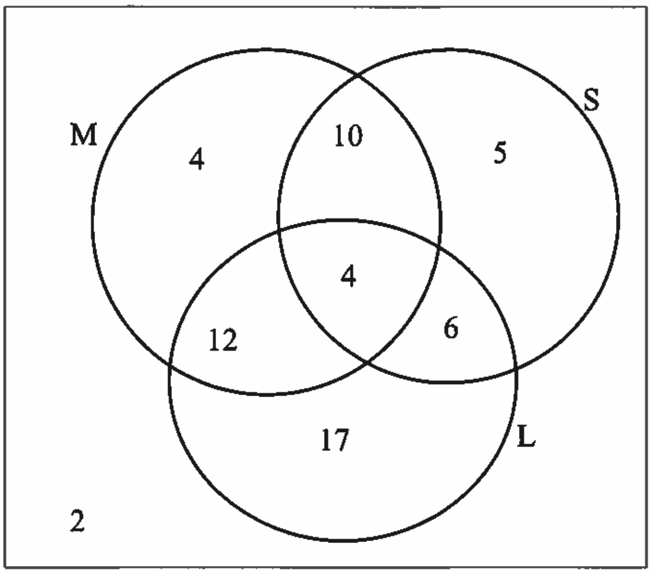
QUESTION 9

9.1		$f(x)=x^3-3x^2-9x+27$ $f^{\prime}(x)=3x^2-6x-9$ $3x^2-6x-9=0$ $3(x-3)(x+1)=0$ $x=3$ or $x=-1$ For maximum turning point, $x=-1$ $\therefore f(-1)=32$ C$(-1\ ;\ 32)$	✓ $f^{\prime}(x)$ ✓ $f^{\prime}(x)=0$ ✓vales of x ✓point C	(4)
9.2		$f^{\prime}(x)=3x^2-6x-9$ $f^{\prime}(-2)=3(-2)^2-6(-2)-9$ $=15$ $y-25=15(x+2)$ $y=15x+55$	✓substitution of $x=-2$ into $f^{\prime}(x)$ ✓answer ✓ $y-25=15(x+2)$ ✓ $y=15x+55$	(4)
9.3		The line $y=k$ intersects with the graph of f at only one point when $k<0$ or $k>32$	✓ $k<0$ ✓ $k>32$	(2)
9.4		$f^{\prime}(x)=3x^2-6x-9$ $f^{\prime\prime}(x)=6x-6$ $6x-6<0$ $x<1$	✓ $f^{\prime\prime}(x)=6x-6$ ✓ $6x-6<0$ ✓answer	(3)
				[13]

QUESTION 10

10.1		Height $(h) = (30-x)$ and radius $(r) = \frac{1}{2}x$ $V(x) = \pi r^2 \times h$ $V(x) = \pi\left(\frac{x}{2}\right)^2 (30-x)$ $= \frac{30\pi x^2}{4} - \frac{\pi x^3}{4}$ $= \frac{15\pi x^2}{2} - \frac{\pi x^3}{4}$	✓$r = \frac{1}{2}x$ ✓substitution into the correct formula	(2)
10.2		$V(x) = \frac{15\pi x^2}{2} - \frac{\pi x^3}{4}$ For maximum volume: $V'(x) = 15\pi x - \frac{3\pi x^2}{4}$ $15\pi x - \frac{3\pi x^2}{4} = 0$ $3\pi x\left(5 - \frac{x}{4}\right) = 0$ $x \neq 0$ or $x = 20$	✓$V'(x)$ ✓$V'(x) = 0$ ✓factors ✓ $x \neq 0$ or $x = 20$	(4)
10.3		$V(x) = \frac{15\pi x^2}{2} - \frac{\pi x^3}{4}$ $V(20) = \frac{15\pi(20)^2}{2} - \frac{\pi(20)^3}{4}$ $= 3141{,}59\ cm^3$	✓substitution of 20 ✓answer	(2)
				[8]

QUESTION 11

11.1	11.1.1		✓M ✓S ✓L ✓2	(4)
	11.1.2	P(a learner takes either of the two subjects but not Mathematics) $=\frac{5}{60}+\frac{6}{60}+\frac{17}{60}$ $=\frac{28}{60}=\frac{7}{15}$	✓ $\frac{5}{60}+\frac{6}{60}+\frac{17}{60}$ ✓ $\frac{28}{60}=\frac{7}{15}$	(2)
11.2		$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,48 + 0,31 - 0,67$ $= 0,12$ But $P(A) \times P(B) = 0,48 \times 0,31 = 0,1488$ $P(A \text{ and } B) \neq P(A) \times P(B)$ $\therefore$ A and B are not independent	✓substitution into the correct formula ✓0,12 ✓0,1488 ✓conclusion	(4)
11.3	11.3.1	Number of arrangements: $\frac{8!}{2! \times 2!} = 10080$	✓✓answer	(2)
	11.3.2	Number of arrangements: $\frac{3!}{2!} \times \frac{6!}{2!} = 1080$ P(vowels will follow each other) $= \frac{1080}{10080} = \frac{3}{28}$	✓ $\frac{3!}{2!}$ ✓ $\frac{6!}{2!}$ ✓answer	(3)
				[15]

TOTAL : 150