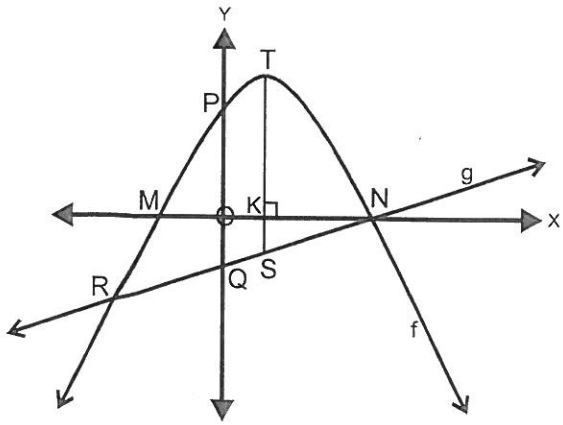
Worksheet 2015-12ℰE-00011(1) Applications

(1) In the sketch:

$f(x) = -x^2 + 2x + 8$ and $g(x) = x - 4$



(a) Determine the length of:

(i) MN
(ii) PQ
(iii) OK and TS

(b) Calculate the coordinates of R.

(c) Write down the coordinates of the turning point of $f(x - 2)$.

(d) $h(x)$ is the reflection of $f(x)$ in the y-axis. Write down the equation of h.

(e) Determine the average gradient of $f(x)$ between $x = 3$ and $x = 5$.

(a)(i) M and N are the x-intercepts of f. ($\therefore y = 0$)

$\therefore 0 = -x^2 + 2x + 8$

$0 = x^2 - 2x - 8$

$0 = (x - 4)(x + 2)$

$x = 4$ or $x = -2$

$N(4; 0)$ $M(-2; 0)$

$\therefore ON = 4$ and $OM = 2$

$\therefore MN = 4 + 2 = 6$

(ii) P and Q are the y-intercepts of f and g.

$\therefore P(0; 8)$ and $Q(0; -4)$

$\therefore OP = 8$ and $OQ = 4$

$\therefore PQ = 8 + 4$

$= 12$

(iii) T is the turning point of T.

$\therefore$ For symmetry axis: $y = -x^2 + 2x + 8$

$x = \frac{-b}{2a}$ or $x = \frac{x_1 + x_2}{2}$

$= \frac{-2}{2(-1)}$ $= \frac{4 + (-2)}{2}$

$x = \frac{-2}{-2} = 1$ $x = \frac{2}{2} = 1$

$\therefore OK = 1$

and $y = -(1)^2 + 2(1) + 8 = -1 + 2 + 8 = 9$

$\therefore TK = 9$

For KS: $x = 1$ in $g(x) = y = x - 4$

$\therefore y = 1 - 4 = -3$

$\therefore KS = 3$

$\therefore TS = 9 + 3 = 12$

(b) R and N are the points of intersection of f and g:

$\therefore -x^2 + 2x + 8 = x - 4$

$0 = x - 4 + x^2 - 2x - 8$

$0 = x^2 - x - 12$

$0 = (x - 4)(x + 3)$

$x = 4$ or $x = -3$

$N(4; 0)$ $\therefore y = x - 4$

$y = -3 - 4$

$y = -7$

$\therefore R(-3; -7)$

(c) $f(x-2)$ moves 2 units to the right

$\therefore$ if $f \rightarrow$ TP $(1;9)$

then $f(x-2) \rightarrow$ TP $(1+2;9)$

$\therefore$ TP $(3;9)$

(d) Reflection in y-axis: $f(x) \rightarrow f(-x)$

$\therefore h(x) = f(-x) = -(-x)^2 + 2(-x) + 8$

$\therefore h(x) = -x^2 - 2x + 8$

(e) $f(x) = -x^2 + 2x + 8$

$\therefore f(3) = -(3)^2 + 2(3) + 8$

$f(3) = -9 + 6 + 8 = 5$ $\therefore (\overset{x_1}{3};\overset{y_1}{5})$

$f(5) = -(5)^2 + 2(5) + 8$

$f(5) = -25 + 10 + 8 = -7$ $\therefore (\overset{x_2}{5};\overset{y_2}{-7})$

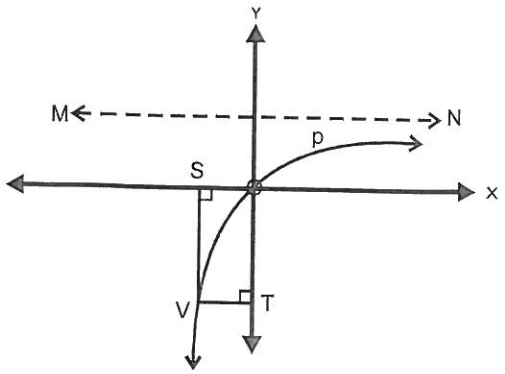
$\therefore AG = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-7 - 5}{5 - 3}$

$= \frac{-12}{2}$

$AG = -6$

(2) In the sketch $p(x) = -2^{-x} + 1$



(a) Write down the equation of MN.

(b) Calculate the length of SV if VT = 1.

(c) Find the equation of $q(x)$ if $q(x)$ is the reflection of p in the y-axis.

(d) Find the equation of $h(x)$ if $h(x)$ the graph of p which translates 2 units upwards.

(e) Write down the range of p.

(a) MN: $y = 1$

(b) $VT = 1 \rightarrow x = -1$

$\therefore y = p(-1) = -2^{-(-1)} + 1$

$= -2^{1} + 1$

$= -2 + 1$

$p(-1) = -1$

$\therefore SV = 1$

(c) Reflection in y-axis $\rightarrow q(x) = p(-x)$

$\therefore q(x) = -2^{-(-x)} + 1$

$\therefore q(x) = -2^{x} + 1$

(d) $h(x) = p(x) + 2$

$= -2^{-x} + 1 + 2$

$\therefore h(x) = -2^{-x} + 3$

(e) R_p: $y < 1$