MEMO 2023-12ƐD-00003(1) Arithmetic series

(1) Calculate:

(a) $5 + 8 + 11 + 14 + \ldots\ldots\ldots$ to 16 terms

$a = 5$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 8 - 5 = 3$ $\qquad S_{16} = \frac{16}{2}[2 \times 5 + (16 - 1)(3)]$

$n = 16$ $\qquad = 8[10 + 15 \times 3]$

$S_n = ?$ $\qquad S_{16} = 440$

(b) $36 + 31 + 26 + 21 + \ldots\ldots\ldots$ to 34 terms

$a = 36$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 31 - 36 = -5$ $\qquad S_{34} = \frac{34}{2}[2 \times 36 + (34 - 1)(-5)]$

$n = 34$ $\qquad = 17[72 + 33 \times -5]$

$S_n = ?$ $\qquad S_{34} = -1\,581$

(c) $9 + 12 + 15 + 18 + \ldots\ldots\ldots + 264$

$a = 9$ $\qquad T_n = a + (n - 1)d$

$d = 3$ $\qquad 264 = 9 + (n-1)(3)$

$n = ?$ $\qquad 264 = 9 + 3n - 3$

$T_n = 264$ $\qquad 258 = 3n$

$S_n = ?$ $\qquad \therefore \quad n = 86$

$\therefore \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{86} = \frac{86}{2}[2 \times 9 + (86 - 1)(3)]$

$= 43[18 + 85 \times 3]$

$S_{86} = 11\,739$ $\qquad$ or $\qquad S_{86} = \frac{86}{2}[9 + 264] = 11\,739$

(d) $\sum_{k=1}^{7} (4k - 1)$

Sequence: $4(1) - 1$; $4(2) - 1$; $4(3) - 1$; …… ; $4(7) - 1$

$= 3$; 7 ; 11 ; …… ; 27

$a = 3$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$ $\qquad$ or $\qquad S_n = \frac{n}{2}[a + \ell]$

$d = 4$ $\qquad S_7 = \frac{7}{2}[2 \times 3 + (7 - 1)(4)]$ $\qquad S_7 = \frac{7}{2}[3 + 27]$

$n = 7$ $\qquad = 3{,}5[6 + 6 \times 4]$ $\qquad S_7 = 105$

$\ell = 27$ $\qquad S_7 = 105$

$S_n = ?$

(e) $-7 - 2 + 3 + 8 + \ldots\ldots\ldots + 123$

$a = -7$ $\quad T_n = a + (n-1)d$ $\quad \therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 5$ $\quad 123 = -7 + (n-1)(5)$ $\quad S_{27} = \frac{27}{2}[2 \times -7 + (27-1)(5)]$

$n = ?$ $\quad 123 = -7 + 5n - 5$ $\quad = 13{,}5[-14 + 26 \times 5]$

$T_n = 123$ $\quad 135 = 5n$ $\quad S_{27} = 1\,566$

$S_n = ?$ $\quad \therefore n = 27$

or $S_{27} = \frac{27}{2}[-7 + 123] = 1\,566$

(f) $\sum_{n=3}^{12} (3 - n)$

Sequence: $3-(3)\ ;\ 3-(4);\ 3-(5);\ \ldots\ldots\ ;\ 3-(12)$

$= 0\ ;\ -1\ ;\ -2\ ;\ \ldots\ldots\ ;\ -9$

$a = 0$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$ $\quad$ or $\quad S_n = \frac{n}{2}[a + \ell]$

$d = -1$ $\quad S_{10} = \frac{10}{2}[2 \times 0 + (10-1)(-1)]$ $\quad S_{10} = \frac{10}{2}[0 + (-9)]$

$n = 10$ $\quad = 5[0 + 9 \times -1]$ $\quad S_{10} = -45$

$\ell = -9$ $\quad S_{10} = -45$

$S_n = ?$

(g) $-66 - 64 - 62 - 60 - \ldots\ldots\ldots - 22$

$a = -66$ $\quad T_n = a + (n-1)d$ $\quad \therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$d = 2$ $\quad -22 = -66 + (n-1)(2)$ $\quad S_{23} = \frac{23}{2}[2 \times -66 + (23-1)(2)]$

$n = ?$ $\quad -22 = -66 + 2n - 2$ $\quad = 11{,}5[-132 + 22 \times 2]$

$T_n = -22$ $\quad 46 = 2n$ $\quad S_{23} = -1\,012$

$S_n = ?$ $\quad \therefore n = 23$

or $S_{23} = \frac{23}{2}[-66 + (-22)] = -1\,012$

(h) n if $\sum_{k=1}^{n}(3k - 2) = 92$

Sequence: $3(1) - 2\ ;\ 3(2) - 2\ ;\ 3(3) - 2\ ;\ \ldots\ldots\ ;\ 3(n) - 2$

$= 1\ ;\ 4\ ;\ 7\ ;\ \ldots\ldots$

$a = 1$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 3$ $\qquad 92 = \frac{n}{2}[2 \times 1 + (n - 1)(3)]$

$n = ?$ $\qquad 184 = n[2 + 3n - 3]$

$S_n = 92$ $\qquad 184 = 2n + 3n^2 - 3n$

$$0 = 3n^2 - n - 184$$

$$0 = (3n + 23)(n - 8)$$

$\therefore\ n = -\frac{23}{3}$ N/a $(n \in \mathbb{N}_0)$ or $n = 8$

(i) $\sum_{i=1}^{300}\left(\frac{i}{2}\right)$

Sequence: $\frac{(1)}{2}\ ;\ \frac{(2)}{2};\ \frac{(3)}{2}\ldots\ldots\ ;\ \frac{(300)}{2}$

$= \frac{1}{2}\ ;\ 1\ ;\ 1\frac{1}{2}\ ;\ \ldots\ldots\ ;\ 150$

$a = \frac{1}{2}$ $\qquad S_{300} = \frac{300}{2}\left[2 \times \frac{1}{2} + (300 - 1)\left(\frac{1}{2}\right)\right]$ or $S_n = \frac{n}{2}[a + \ell]$

$d = \frac{1}{2}$ $\qquad S_{300} = \frac{300}{2}\left[1 + (299)\left(\frac{1}{2}\right)\right]$ $\qquad S_{300} = \frac{300}{2}\left[\frac{1}{2} + 150\right]$

$n = 300$ $\qquad = 150\left[1 + (299)\left(\frac{1}{2}\right)\right]$ $\qquad S_{300} = 22\,575$

$\ell = 150$ $\qquad S_{300} = 22\,575$

$S_n = ?$

(j) n if $0{,}3 + 1{,}1 + 1{,}9 + 2{,}7 + \ldots\ldots\ldots$ (to n terms) $= 24{,}8$

$a = 0{,}3$ $\qquad S_n = \frac{n}{2}[2a + (n - 1)d]$

$d = 0{,}8$ $\qquad 24{,}8 = \frac{n}{2}[2 \times 0{,}3 + (n - 1)(0{,}8)]$

$n = ?$ $\qquad 49{,}6 = n[0{,}6 + 0{,}8n - 0{,}8]$

$S_n = 24{,}8$ $\qquad 49{,}6 = 0{,}6n + 0{,}8n^2 - 0{,}8n$

$$0 = 0{,}8n^2 - 0{,}2n - 49{,}6 \qquad [\times 10]$$

$$0 = 8n^2 - 2n - 496$$

$$0 = 4n^2 - n - 248$$

$$0 = (4n + 31)(n - 8)$$

$\therefore\ n = \frac{-31}{4}$ N/a $(n \in \mathbb{N}_0)$ or $n = 8$

(2) The n^{th} term of an AS is $2n + 3$. Determine:

(a) the first three terms of the sequence.

$$T_n = 2n + 3$$
$$T_1 = 2(1) + 3 = 2 + 3 = 5$$
$$T_2 = 2(2) + 3 = 4 + 3 = 7$$
$$T_3 = 2(3) + 3 = 6 + 3 = 9$$
$$\therefore\ 5\ ;\ 7\ ;\ 9$$

(b) the 18^{th} term of the sequence.

$$T_n = 2n + 3$$
$$T_{18} = 2(18) + 3$$
$$T_{18} = 36 + 3$$
$$\therefore\ T_{18} = 39$$

(c) how many terms in the sequence have a sum of 4 352.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$
$$4\,352 = \frac{n}{2}[2(5) + (n - 1)(2)]$$
$$8\,704 = n[10 + 2n - 2]$$
$$8\,704 = 8n + 2n^2$$
$$0 = 8n + 2n^2 - 8\,704$$
$$0 = 2n^2 + 8n - 8\,704$$
$$0 = n^2 + 4n - 4\,352$$
$$0 = (n + 68)(n - 64)$$
$$n + 68 = 0 \quad \text{or} \quad n - 64 = 0$$
$$\therefore\ n = -68\ \text{N/a}\ (n \in \mathbb{N}_0) \qquad \text{or} \qquad n = 64$$

(3) The following is given: $\sum_{t=2}^{11} (3 - 3t)$

(a) Write down the first three terms.

$T_1 = 3 - 3(2) = 3 - 6 = -3 \quad \rightarrow \quad T_1 = -3$

$T_2 = 3 - 3(3) = 3 - 9 = -6 \quad \rightarrow \quad T_2 = -6$

$T_3 = 3 - 3(4) = 3 - 12 = -9 \quad \rightarrow \quad T_3 = -9$

(b) Determine the sum of the series.

$a = -3$

$d = -3$

$n = 10$

$S_n = ?$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2 \times -3 + (10 - 1)(-3)]$$

$$= 5[-6 + 9 \times -3]$$

$$= 5[-6 - 27]$$

$$S_{10} = -165$$