NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2024

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

<table>
<tr><td>(a)(1)</td><td>

Age	22	24	27	30	35	39	43	45
bpm	180	150	120	180	110	90	110	200

$y = 171{,}983 - 0{,}890x$</td><td>Use of correct values from graph
171,983
0,890</td></tr>
<tr><td>(a)(2)</td><td>$y = 171{,}983 - 0{,}890(50)$
$y \approx 127{,}5$ bpm</td><td>$y = 171{,}983 - 0{,}890(50)$
$y \approx 127{,}5$ bpm</td></tr>
<tr><td>(a)(3)</td><td>Extrapolation – 50 is outside of the given range
There is a weak correlation of –0,2</td><td>Extrapolation – 50 is outside
weak correlation of –0,2</td></tr>
<tr><td>(b)(1)</td><td>$60 \times 25\%$
$\therefore 15$ students</td><td>$\therefore 15$ students</td></tr>
<tr><td>(b)(2)</td><td>$Q_1 - 1{,}5 \times \text{IQR}$
$= 27 - 1{,}5 \times 6$
$= 18$
It is not an outlier since $24 > 18$</td><td>$= 18$
Not an outlier</td></tr>
<tr><td>(b)(3)</td><td>No effect on the median</td><td>No effect</td></tr>
</table>

QUESTION 2

(a)	$m_{BC} = \frac{2}{3}$ BC ∥ AO $y = \frac{2}{3}x + c$ ... sub. $(-3;6)$ $c = 8$ $\therefore y = \frac{2}{3}x + 8$ **Alternate:** $m_{BC} = \frac{2}{3}$ BC ∥ AO $y - 6 = \frac{2}{3}(x+3)$ $\therefore y = \frac{2}{3}x + 8$	$m_{BC} = \frac{2}{3}$ $y = \frac{2}{3}x + c$ $c = 8$ $\therefore y = \frac{2}{3}x + 8$
(b)	Sub. $x = -3$ in $y = \frac{2}{3}x$ $y = -2$ $A(-3;-2)$	$x = -3$ $x = -3$ in $y = \frac{2}{3}x$ $y = -2$ $A(-3;-2)$
(c)	$m_{AB} = m_{OC}$ ∥ lines $m_{OC} = \frac{0-6}{0+3}$ $\therefore m_{OC} = -2$ $y = -2x + c$ ... sub. $(-3;-2)$ $c = -8$ $\therefore y = -2x - 8$ **Alternate:** $m_{AB} = m_{OC}$ ∥ lines $m_{OC} = \frac{0-6}{0+3}$ $\therefore m_{OC} = -2$ $y + 2 = -2(x+3)$ $\therefore y = -2x - 8$	$m_{OC} = -2$ $y = -2x + c$ $c = -8$ $y = -2x - 8$

(d)	Point of intersection of BC and AB: $\frac{2}{3}x+8=-2x-8$ $x=-6$ $\therefore y=4$ $B(-6;4)$ **Alternate:** By symmetry: move point C – 3 units left and 2 units down. $B(-6;4)$	$\frac{2}{3}x+8=-2x-8$ $x=-6$ $\therefore y=-2(-6)-8$ $\therefore y=4$

QUESTION 3

(a)	$\hat{B} = 90°$ … angle in semicircle $\therefore x = 114°$ …ext. angle of triangle	$\hat{B} = 90°$ angle in semicircle $\therefore x = 114°$
(b)	$\hat{D}_1 = 24°$ … angles in same segment	$\hat{D}_1 = 24°$ angles in same segment
(c)	$C\hat{E}B = 2 \times 24°$ … angle at centre = 2X $= 48°$	$C\hat{E}B = 2 \times 24°$ $= 48°$ angle at centre = 2X

QUESTION 4

(a)	Construct: AF through centre O. Join BF. $D\hat{A}F = 90°$ … tangent perp. to radius $\therefore B\hat{A}F = 90° - x$ $A\hat{B}F = 90°$ … angle in semicircle $\therefore$ In ΔABF: $\hat{F} = x$ … int angles of triangle $\therefore \hat{C} = y = x$ … angles in same segment AB Alternative: Construct: AF through centre O. Join CF. $D\hat{A}F = 90°$ … tangent perp. to radius $\therefore B\hat{A}F = 90° - x = B\hat{C}F$ … angles in same segment But, $A\hat{C}F = 90°$ … angle in semicircle Therefore, x=y	AF through centre O. Join BF. $D\hat{A}F = 90°$ … tangent perp. to radius $\therefore B\hat{A}F = 90° - x$ $A\hat{B}F = 90°$ … angle in semi-circle ΔABF: $\hat{F} = x$ $\therefore \hat{C} = y = x$ … angles in same segment AB
(b)(1)	$O\hat{B}C = 90°$ … tangent perp. to radius $\therefore \hat{B}_1 = 90° - 38°$ $= 52°$ $\therefore \hat{A} = 52°$ … radii/angles opp. equal sides	$O\hat{B}C = 90°$ tangent perp. to radius $\therefore \hat{B}_1 = 52°$ $\therefore \hat{A} = 52°$ radii/angles opp. equal sides
(b)(2)	$\therefore \hat{O}_3 = 180° - (2 \times 52°)$ … int angles of triangle $= 76°$ $\therefore \hat{O}_1 = 76°$ … equal chords subtend equal angles	$\therefore \hat{O}_3 = 76°$ $\therefore \hat{O}_3 = 76°$ equal chords subtend equal angles

QUESTION 5

(a)	$D\hat{E}B = E\hat{B}C$ … alt angles DE\\BC $E\hat{D}C = D\hat{C}B$ … alt angles DE\\BC $B\hat{T}C = D\hat{T}E$ … Vertically opp. angles $\therefore \Delta DET /// \Delta CBT$ … (AAA)	$D\hat{E}B = E\hat{B}C$ … alt angles DE\\BC $E\hat{D}C = D\hat{C}B$ … alt angles DE\\BC $\therefore \Delta DET /// \Delta CBT$ … (AAA)
(b)	$\frac{DE}{BC} = \frac{DT}{CT}$ … /// triangles; sides in prop $\frac{8}{BC} = \frac{4}{7}$ $BC = 14$ $\frac{ET}{BT} = \frac{DT}{CT}$ $\frac{6}{BT} = \frac{4}{7}$ $BT = 10\frac{1}{2}$ $\therefore BC + BT = 24\frac{1}{2}$	$\frac{DE}{BC} = \frac{DT}{CT}$ … /// triangles; sides in prop $\frac{8}{BC} = \frac{4}{7}$ $BC = 14$ $\frac{6}{BT} = \frac{4}{7}$ $BT = 10\frac{1}{2}$ $\therefore BC + BT = 24\frac{1}{2}$
(c)	$\Delta ADE /// \Delta ABC$ … equiangular $\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{AD}{AB} = \frac{8}{14} = \frac{AE}{AC}$ $\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{4}{7}$ … line \\ one side of triangle $\therefore AD : DB = 4 : 3$	$\Delta ADE /// \Delta ABC$ … equiangular $\therefore \frac{AD}{AB} = \frac{8}{14} = \frac{AE}{AC}$ $\therefore AD : DB = 4 : 3$

QUESTION 6

(a)	$\frac{(\sin\theta)(\cos\theta)(-\tan\theta)}{(\sin 2\theta)(\tan\theta)}$ $=\frac{(\sin\theta)(\cos\theta)(-\tan\theta)}{(2\sin\theta\cos\theta)(\tan\theta)}$ $=-\frac{1}{2}$	$\sin\theta$ $\cos\theta$ $-\tan\theta$ $\sin 2\theta$ $\tan\theta$ $2\sin\theta\cos\theta$ $=-\frac{1}{2}$
(b)(1)	$\sin 2(17°)$ $=2\sin 17°.\cos 17°$ $=2k$	$=2\sin 17°.\cos 17°$ $=2k$
(b)(2)	$-(1-2\sin^2 17°)$ $=-\cos 2(17°)$ $=-\cos 34°$ $\sin 34° = \frac{2k}{1} = \frac{y}{r}$ $x=\sqrt{1-4k^2}$ … pythag. $\therefore -\cos 34° = -\sqrt{1-4k^2}$	$-(1-2\sin^2 17°)$ $=-\cos 2(17°)$ $=-\cos 34°$ $x=\sqrt{1-4k^2}$ $\therefore -\cos 34° = -\sqrt{1-4k^2}$
(c)(1)	$x^2 = 3^2 - 2^2$ … pythag $x=-\sqrt{5}$ $\tan(-A)$ $=-\tan A$ $=\frac{2}{\sqrt{5}}$	$x^2 = 3^2 - 2^2$ $x=-\sqrt{5}$ $=-\tan A$ $=\frac{2}{\sqrt{5}}$
(c)(2)	$\sin A.\cos 30° + \cos A.\sin 30°$ $=\left(\frac{2}{3}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{-\sqrt{5}}{3}\right)\left(\frac{1}{2}\right)$ $=\frac{2\sqrt{3}-\sqrt{5}}{6}$	$\sin A.\cos 30° + \cos A.\sin 30°$ $\left(\frac{2}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$ $\left(\frac{-\sqrt{5}}{3}\right)\left(\frac{1}{2}\right)$ $\frac{2\sqrt{3}-\sqrt{5}}{6}$

SECTION B

QUESTION 7

<table>
<tr><td>(a)</td><td>$x^2 - 10x + y^2 + 14y = -49$
$(x-5)^2 + (y+7)^2 = 25$

Circle Centre: (5; –7)
T (2; –3) lies on the circle
$m_{PT} = \frac{-3+7}{2-5} = -\frac{4}{3}$
$\therefore m = \frac{3}{4}$
Eq of tangent: $y = \frac{3}{4}x + c$
Sub: (2; –3)

$\therefore c = -\frac{9}{2}$
$y = \frac{3}{4}x - \frac{9}{2}$
$\therefore 4y = 3x - 18$
$\therefore k = 3 \text{ and } t = -18$

Alternate to equation of tangent:

$y - 3 = \frac{3}{4}(x-2)$</td><td>$(x-5)^2 + (y+7)^2 = 25$
Circle Centre: $(5;-7)$
$m_{PT} = \frac{-3+7}{2-5} = -\frac{4}{3}$
$m_{\tan} = \frac{3}{4}$
$c = -\frac{9}{2}$
$y = \frac{3}{4}x - \frac{9}{2}$
$k = 3$
$t = -18$</td></tr>
<tr><td>(b)</td><td>Sub: (7; 0) in second circle equation to determine r.
$(x-4)^2 + (y-3)^2 = r^2$
$(7-4)^2 + (0-3)^2 = r^2$
$r = \sqrt{18}$

Dist between centres:
$\sqrt{(4-5)^2 + (3+7)^2}$
$= \sqrt{101}$
≈ 10

Sum of radii = $5 + \sqrt{18}$
$\approx 9{,}2$

Dist between centres > sum of radii
Therefore, the circles do not intersect.</td><td>$(7-4)^2 + (0-3)^2 = r^2$
$r = \sqrt{18}$
$= \sqrt{101}$
$5 + \sqrt{18}$
Dist between centres > sum of radii

do not intersect</td></tr>
</table>

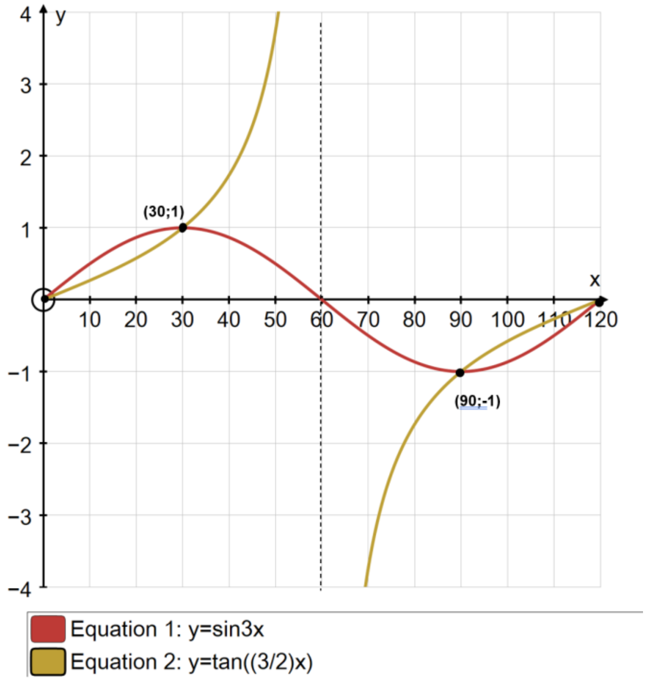
QUESTION 8

(a)	$\tan C\hat{A}B = 3$ $\therefore C\hat{A}B = 71{,}565°$ $\therefore C\hat{B}A = 71{,}565° \ldots \angle$ opp $=$ sides $m_{DB} = \tan(180° - 71{,}565°)$ $= -3$ Equation BD: $y = -3x + c$ Sub. $(6;3)$ $\therefore c = 21$ $\therefore y = -3x + 21$	$\tan C\hat{A}B = 3$ $C\hat{B}A = 71{,}565°$ $m_{DB} = \tan(180° - 71{,}565°)$ $= -3$ $y = -3x + c$ $\therefore c = 21$ $\therefore y = -3x + 21$
(b)	For co-ordinates of A: $m_{AC} = 3 = \dfrac{3-0}{6-x}$ $\therefore x = 5$ $A(5;0)$ and $D(0;21)$ For centre of circle: Midpoint DA$\left(\dfrac{5}{2};\dfrac{21}{2}\right)$ Equation of circle: $\left(x-\dfrac{5}{2}\right)^2 + \left(y-\dfrac{21}{2}\right)^2 = r^2$ Sub.: $(5;0)$ $\therefore r^2 = 116{,}5$ $\left(x-\dfrac{5}{2}\right)^2 + \left(y-\dfrac{21}{2}\right)^2 = 116{,}5$	$x = 5$ Midpoint DA$\left(\dfrac{5}{2};\dfrac{21}{2}\right)$ $\left(x-\dfrac{5}{2}\right)^2 + \left(y-\dfrac{21}{2}\right)^2 = r^2$ Sub.: $(5;0)$ $\therefore r^2 = 116{,}5$ $\left(x-\dfrac{5}{2}\right)^2 + \left(y-\dfrac{21}{2}\right)^2 = 116{,}5$

QUESTION 9

(a)	$\sin 2\left(\frac{3}{2}\right)x = \dfrac{\sin\frac{3}{2}x}{\cos\frac{3}{2}x}$ $2\sin\frac{3}{2}x.\cos\frac{3}{2}x = \dfrac{\sin\frac{3}{2}x}{\cos\frac{3}{2}x}$ $2\sin\frac{3}{2}x.\cos\frac{3}{2}x - \sin\frac{3}{2}x = 0$ $\therefore \sin\frac{3}{2}x\left(2\cos^2\frac{3}{2}x - 1\right) = 0$ $\therefore \sin\frac{3}{2}x\left(2\cos 3x\right) = 0$ $\therefore \sin\frac{3}{2}x = 0 \;\; or \;\; \cos 3x = 0$ $\frac{3}{2}x = 180°k \quad \therefore x = 120°k$ or $3x = \pm 90° + 360°k$ $\therefore x = \pm 30° + 120°k$ $\therefore x = 120°k \;\; or \;\; x = 30° + 60°k;\; k \in \mathbb{Z}$	$\sin 2\left(\frac{3}{2}\right)x = \dfrac{\sin\frac{3}{2}x}{\cos\frac{3}{2}x}$ $2\sin\frac{3}{2}x.\cos\frac{3}{2}x$ $\therefore \sin\frac{3}{2}x\left(2\cos^2\frac{3}{2}x - 1\right) = 0$ $(\cos 3x)$ $\frac{3}{2}x = 360°k$ or $\frac{3}{2}x = 180° + 360°k$ $x = 240°k$ or $x = 120°k + 240°k$ $\therefore x = \pm 30° + 120°k$
	Alternative: $\sin 3x = \tan\frac{3}{2}x$ $Let: \frac{3}{2}x = p$ $\sin 2p = \tan p$ $\sin 2p = \dfrac{\sin p}{\cos p}$ $2\sin p\cos^2 p = \sin p$ $\sin p\left(2\cos^2 p - 1\right) = 0$ $\sin p\left(\cos 2p\right) = 0$ $\sin p = 0 \;\; or \;\; \cos 2p = 0$ $\sin p = 0$ $\therefore p = 180°k$ $\therefore \frac{3}{2}x = 180°k$ $\therefore x = 120°k$ or $\cos 2p = 0$ $\therefore 2p = 90° + 180°k$ $\therefore 3x = 90° + 180°k$ $\therefore x = 30° + 60°k$ $\therefore x = 120°k \;\; or \;\; x = 30° + 60°k;\; k \in \mathbb{Z}$	$\sin 2p = \dfrac{\sin p}{\cos p}$ $2\sin p\cos^2 p = \sin p$ $\sin p\left(2\cos^2 p - 1\right) = 0$ $\sin p = 0$ $x = 120°k$ or $\cos 2p = 0$ $\therefore 3x = 90° + 180°k$ $\therefore x = 30° + 60°k$

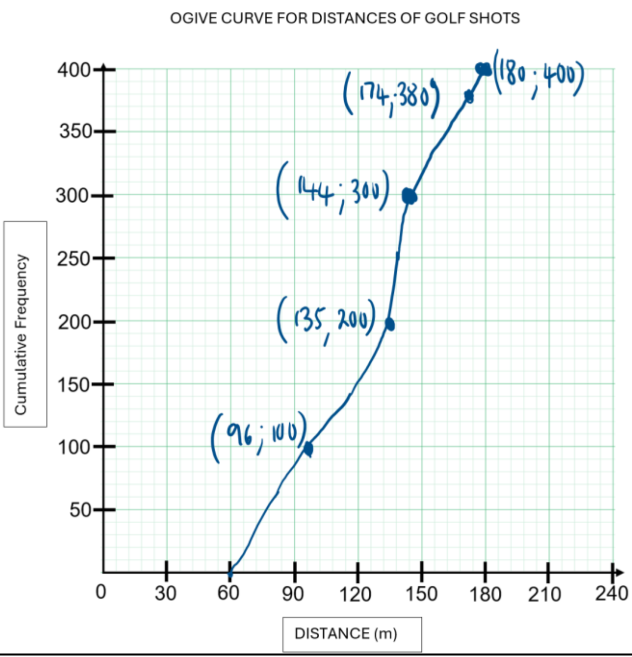
	Alternate: $\sin 3x = \tan\frac{3}{2}x$ $Let: \frac{3}{2x} = p$ $\sin 2p = \tan p$ $\sin 2p = \frac{\sin p}{\cos p}$ $2\sin p\cos^2 p = \sin p$ $\sin p\left(2\cos^2 p - 1\right) = 0$ $\sin p = 0 \ or \ 2\cos^2 p - 1 = 0$ $\sin p = 0$ $\therefore p = 180°k$ $\therefore \frac{3}{2}x = 180°k$ $\therefore x = 120°k$ or $2\cos^2 p - 1 = 0$ $\therefore \cos^2 p = \frac{1}{2}$ $\cos p = \pm\sqrt{\frac{1}{2}}$ $\therefore \frac{3}{2}x = 45° + 90°k$ $\therefore x = 30° + 60°k \quad k \in \mathbb{Z}$ $\therefore x = 120°k \ or \ x = 30° + 60°k; k \in \mathbb{Z}$	$\sin 2p = \frac{\sin p}{\cos p}$ $2\sin p\cos^2 p = \sin p$ $\sin p\left(2\cos^2 p - 1\right) = 0$ $\sin p = 0$ $x = 120°k$ $\cos p = \pm\sqrt{\frac{1}{2}}$ $p = 45° + 90°k$ $\therefore \frac{3}{2}x = 45° + 90°k$ $\therefore x = 30° + 60°k$

(b)		Graph of f: Intercepts amplitude period Graph of g asymptote intercepts intersections
(c)	$\{0^\circ \le x \le 30^\circ\} \cup \{60^\circ < x \le 90^\circ\}$	$\{0^\circ \le x \le 30^\circ\}$ $\{60^\circ < x \le 90^\circ\}$

QUESTION 10

(a)	$2\cos^2\theta - 1 + 3\left(2\sin\theta.\cos\theta\right) = -1$ $2\cos^2\theta - 1 + 6\sin\theta.\cos\theta + 1 = 0$ $2\cos\theta\left(\cos\theta + 3\sin\theta\right) = 0$ $\cos\theta + 3\sin\theta = 0$ $\tan\theta = -\frac{1}{3}$ $\therefore \theta = -18,4^\circ + 180^\circ k \quad ; \quad k \in Z$ $\therefore \theta = \left\{-18,4^\circ; -198,4^\circ\right\}$	$2\cos^2\theta$ -1 $2\sin\theta\cos\theta$ $cos\,\theta + 3\,sin\,\theta = 0$ $\tan\theta = -\frac{1}{3}$ $\theta = -18,4^\circ + 180^\circ k$ $\theta = \{-18,4^\circ;\ -198,4^\circ\}$
(b)	$\sqrt{\cos 9\theta - 2\cos 4\theta.\cos 5\theta}$ $= \sqrt{\cos(4\theta + 5\theta) - 2\cos 4\theta.\cos 5\theta}$ $= \sqrt{\cos 4\theta.\cos 5\theta - \sin 4\theta.\sin 5\theta - 2\cos 4\theta.\cos 5\theta}$ $= \sqrt{-\cos 4\theta.\cos 5\theta - \sin 4\theta.\sin 5\theta}$ $= \sqrt{-(\cos 4\theta - 5\theta)}$ $= \sqrt{-\cos(-\theta)}$ $= \sqrt{-\cos\theta}$ Valid for $cos\,\theta \leq 0$ $\therefore$ Quad 2 and Quad 3 $\therefore 90^\circ \leq \theta \leq 270^\circ$	$\sqrt{\cos(4\theta + 5\theta) - 2\cos 4\theta.\cos 5\theta}$ $\sqrt{\cos 4\theta.\cos 5\theta - \sin 4\theta.\sin 5\theta - 2\cos 4\theta.\cos 5\theta}$ $\sqrt{-\cos(-\theta)}$ Valid for $cos\,\theta \leq 0$ $\theta \in [90^\circ; 270^\circ]$

QUESTION 11

(a)		(60;0) and (180;400) (96;100) (135; 200) (144; 300) (174; 380)
(b)	No, The distance is close to 120 not 140. **Alternate:** No, 140 is bigger than the median	The distance is close to 120 not 140.
(c)	Since the mean<median, the data is negatively skewed.	mean<median negatively skewed.

QUESTION 12

	In ΔABC: $B\hat{A}C + \hat{B} + \hat{C} = 180°$ ... int $\angle$ of Δ $\hat{F}_3 = \hat{B}$ ... tan-chord theorem $\hat{F}_2 = \hat{C}$ ... tan-chord theorem $\hat{F}_3 + \hat{F}_2 = E\hat{F}D$ $\therefore E\hat{F}D + B\hat{A}C = 180°$ Therefore, AEFD is a cyclic quadrilateral; Converse of opposite angles of cyclic quad	$B\hat{A}C + \hat{B} + \hat{C} = 180°$ $\hat{F}_3 = \hat{B}$ ... tan-chord th $\hat{F}_2 = \hat{C}$ ... tan-chord th $\hat{F}_3 + \hat{F}_2 = E\hat{F}D$ $\therefore E\hat{F}D + D\hat{A}E = 180°$ Therefore, AEFD is a cyclic quadrilateral; Converse of opposite angles of cyclic quad

QUESTION 13

(a)	$AF = 8$ ... tan from point $FC = 21$ ... tan from point $\therefore AC = 29$ $BD = 8$ since D is the midpt $AB = 16$ units $BE = 8$ units ... tan from point $\therefore BC = 29$ units $\cos A\hat{B}C = \frac{29^2 + 16^2 - 29^2}{2 \times 29 \times 16}$ $A\hat{B}C = 73{,}9866^\circ$ $\approx 74^\circ$	$AF = 8$ tan from point $AB = 16$ units tan from point $BC = 29$ units $A\hat{B}C = 73{,}9866^\circ$ $\approx 74^\circ$
(b)	Area $\Delta BOC = \frac{1}{2} \times 29 \times r$ ... tan $\perp$ rad Area $\Delta AOB = \frac{1}{2} \times 16 \times r$ Area $\Delta AOC = \frac{1}{2} \times 29 \times r$ Area $\Delta ABC = \frac{1}{2} \times 16 \times 29 \times \sin 73{,}9866^\circ$ $\approx 222{,}9977516$ $\frac{1}{2} r (29 + 16 + 29) = 222{,}99516$ $\therefore r = 6$ units **Alternate:** $\hat{A} = \hat{B} = 74^\circ$ ... AC = BC from Q13a but $\Delta AFO \equiv \Delta ADO$ (S;S;S) Thus: $\hat{A}_1 = \hat{A}_2 = 37^\circ$ and since $OF \perp AF$ (tan $\perp$ rad) $\frac{r}{8} = \tan 37^\circ$ $\therefore r = 6$ units	Area $\Delta BOC = \frac{1}{2} \times 29 \times r$ ... tan $\perp$ rad Area $\Delta AOB = \frac{1}{2} \times 16 \times r$ Area $\Delta AOC = \frac{1}{2} \times 29 \times r$ Area $\Delta ABC = \frac{1}{2} \times 16 \times 29 \times \sin 73{,}9866^\circ$ $\approx 222{,}9977516$ $\frac{1}{2} r (29 + 16 + 29) = 222{,}99516$ $\therefore r = 6$ units

Total: 150 marks