Agtergrondkennis – **Differensiasie**:

A1 **Limiete**:

Vb.1 Bepaal: (a) $\lim_{x \to -1}(2x + 1)$ = *[2(-1) + 1] =* ***-1*** *Vervang x met -1*

(b) $\lim_{x \to 3}\left(\frac{x^2 - 9}{x - 3}\right)$ *Kan nou al nie x met 3 vervang nie,*

$= \lim_{x \to 3}\left(\frac{(x - 3)(x + 3)}{(x - 3)}\right)$ *want (3 – 3 = 0) en noemer mag ≠ 0.*

$= \lim_{x \to 3}\left(\frac{\cancel{(x - 3)}(x + 3)}{\cancel{(x - 3)}}\right)$ *∴ Faktoriseer eers en vereenvoudig.*

$= \lim_{x \to 3}(x + 3) = 3 + 3 = \mathbf{6}$

A2 **Differensiasie vanuit eerste beginsels**:

Formule vir eerste beginsels:

$$f'(x) = \lim_{h \to 0}\frac{f(x + h) - f(x)}{h}$$

Vb.2 Bepaal $f'(x)$ *as* $f(x) = 2x^2 - x$

****

$f(x) = 2x^2 - x$

$\therefore\ f(x + h) = 2(x + h)^2 - (x + h)$

$\therefore\ f(x + h) = 2(x^2 + 2xh + h^2) - (x + h)$

$\therefore\ f(x + h) = 2x^2 + 4xh + 2h^2 - x - h$

maar $f'(x) = \lim_{h \to 0}\frac{f(x + h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0}\frac{(2x^2 + 4xh + 2h^2 - x - h) - (2x^2 - x)}{h}$

$= \lim_{h \to 0}\frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h - \cancel{2x^2} + \cancel{x}}{h}$

$= \lim_{h \to 0}\frac{4xh + 2h^2 - h}{h}$

$= \lim_{h \to 0}\frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}}$

$= \lim_{h \to 0}(4x + 2h - 1)$

$= 4x + 2(0) - 1 = \mathbf{4x - 1}$

A3 Differensiasiereëls:
A3.1 Reëls:

* Basisreël: $\frac{d}{dx}(x^n) = nx^{n-1}$

* Afgeleide van 'n konstante: $\frac{d}{dx}(k) = 0$ waar *k* enige konstante waarde is

* Afgeleide van die som of verskil van twee funksies:

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

* Afgeleide van 'n konstante vermenigvuldig met 'n funksie:

$$\frac{d}{dx}[k \times f(x)] = k\frac{d}{dx}[f(x)]$$ waar *k* enige konstante waarde is

A3.2 Notasies:

Vb.3 (a) Bepaal $f'(x)$ *as* $f(x) = \frac{5}{x} - 3x + 2x^5 - 1$

$$\therefore f(x) = 5x^{-1} - 3x + 2x^5 - 1$$

$$\therefore f'(x) = -5x^{-2} - 3 + 10x^4$$

$$\therefore \boldsymbol{f'(x) = \frac{-5}{x^2} - 3 + 10x^4}$$

(b) Bepaal $D_x[(3x + m)^2]$

$$= D_x[3x^2 + 6mx + m^2]$$

$$= \boldsymbol{6x + 6m}$$ *[m word hanteer as 'n konstante]*

(c) Bepaal $\frac{dy}{dx}$ *as* $y = \sqrt[3]{8x^2} + \frac{x^2 - 2x - 8}{x - 4}$

$$\therefore y = \sqrt[3]{8} \times \sqrt[3]{x^2} + \frac{(x-4)(x+2)}{(x-4)}$$

$$\therefore y = 2 \times x^{\frac{2}{3}} + \frac{(x-4)(x+2)}{(x-4)}$$

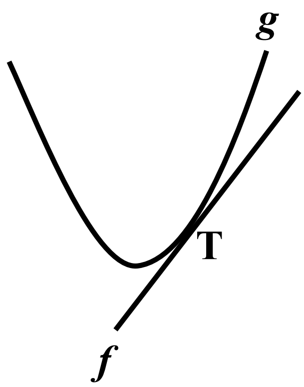
$$\therefore y = 2x^{\frac{2}{3}} + \frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}}$$

$$\therefore y = 2x^{\frac{2}{3}} + x + 2$$

$$\therefore \frac{dy}{dx} = 2 \times \frac{2}{3}x^{\frac{2}{3}-1} + 1$$

$$\therefore \boldsymbol{\frac{dy}{dx} = \frac{4}{3}x^{\frac{-1}{3}} + 1}$$

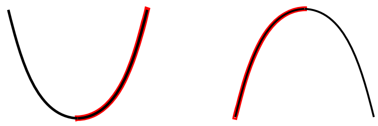
A4 Toepassings:
A4.1 Basiese toepassings:



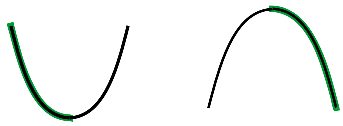
* Om die vergelyking van die raaklyn aan enige kurwe te bereken:

  f(x) = $y - y_1 = m(x - x_1)$
  met T $(x_1 ; y_1)$ die raakpunt en
  m = g$'(x_1)$.

* 'n Funksie is stygend as die gradiënt positief is.



* 'n Funksie is dalend as die gradiënt negatief is.



* 'n Funksie is nie stygend of dalend as die gradiënt 0 is nie.

Vb.4 Bereken die koördinate van S, as S die raakpunt is van raaklyn $y = 3x - 7$ aan die kromme $f(x) = x^2 + x - 6$.

****

Raaklyn: $y = 3x - 7 \quad \rightarrow \quad m = 3$

Maar $m = f'(x) = 2x + 1$

$\therefore \quad 3 = 2x + 1$

$\therefore \quad 3 = 2x + 1$

$\therefore \quad 2 = 2x$

$\therefore \quad x = 1$

Vervang $x = 1$ *in die raaklyn of in f(x):*

$y = 3x - 7$	*of*	$f(x) = x^2 + x - 6$
$y = 3(1) - 7$		$y = (1)^2 + (1) - 6$
$y = -4$		$y = 1 + 1 - 6 = -4$

$\therefore$ ***S (1 ; -4)***

A4.2 Grafieke:
A4.2.1 Oplos van derdegraadse vergelykings:

∴ Verskillende voorbeelde van hoe om die x-afsnit te bepaal: (y = 0)

(1) As $f(x) = x^3 + 2x^2 + x$

x-afsnitte: ($y = 0$) - [Haal x uit as gemeenskaplike faktor.]

∴ $0 = x^3 + 2x^2 + x$

∴ $0 = x(x^2 + 2x + 1)$

∴ $0 = x(x + 1)(x + 1)$

∴ $\boldsymbol{x = 0}$ of $\boldsymbol{x = -1}$ of $\boldsymbol{x = -1}$

(2) As $f(x) = (1 - 2x)(x + 3)(x + 2)$

x-afsnitte: ($y = 0$) - [Faktore gegee.]

∴ $0 = (1 - 2x)(x + 3)(x + 2)$

∴ $1 - 2x = 0$ of $x + 3 = 0$ of $x + 2 = 0$

∴ $\boldsymbol{x = \frac{1}{2}}$ of $\boldsymbol{x = -3}$ of $\boldsymbol{x = -2}$

(3) As $f(x) = x^3 + 2x^2 - x - 2$

x-afsnitte: ($y = 0$) - [Faktoriseer deur te groepeer.]

∴ $0 = x^3 + 2x^2 - x - 2$

∴ $0 = x^2(x + 2) - 1(x + 2)$

∴ $0 = (x + 2)(x^2 - 1)$

∴ $0 = (x + 2)(x - 1)(x + 1)$

∴ $\boldsymbol{x = -2}$ of $\boldsymbol{x = 1}$ of $\boldsymbol{x = -1}$

(4) As $f(x) = -4x^3 + 28x^2 - 49x + 18$

<u>x-afsnitte</u>: ($y = 0$) - [Faktorstelling en sintetiese deling.]

$\therefore\ f(1) = -4(1)^3 + 28(1)^2 - 49(1) + 18 = 91$

$\therefore\ x - 1$ is nie 'n faktor van $f(x)$ nie.

$\therefore\ f(2) = -4(2)^3 + 28(2)^2 - 49(2) + 18 = 0$

$\therefore\ (x - 2)$ is 'n faktor van $f(x) \longrightarrow x = \mathbf{2}$ is die een x-afsnit

$$\begin{array}{c|cccc}
 & -4 & +28 & -49 & +18 \\
2 & 0 & -8 & +40 & -18 \\
\hline
 & -4 & +20 & -9 & 0
\end{array}$$

$\longrightarrow$ koëffisiënte van $f(x)$

$\longrightarrow$ koëffisiënte van ( )

Altyd 0

$\therefore\ 0 = -4x^3 + 28x^2 - 49x + 18$

$\therefore\ 0 = (x - 2)(-4x^2 + 20x - 9)$

$\therefore\ x - 2 = 0$ of $-4x^2 + 20x - 9 = 0$

$\therefore\ \boldsymbol{x = 2}$ of $4x^2 - 20x + 9 = 0$

$\therefore\ (2x - 1)(2x - 9) = 0$

$\therefore\ \boldsymbol{x = \frac{1}{2}}$ of $\boldsymbol{x = \frac{9}{2}}$

(5) As $f(x) = x^3 - 5x^2 + 7x - 3$

<u>x-afsnitte</u>: ($y = 0$) - [Faktorstelling en inspeksie.]

$\therefore\ 0 = x^3 - 5x^2 + 7x - 3$

$\therefore\ f(1) = (1)^3 - 5(1)^2 + 7(1) - 3 = 0$

$\therefore\ (x - 1)$ is 'n faktor van $f(x)$

$\therefore\ x^3 - 5x^2 + 7x - 3 = (x - 1)(x^2 \ldots\ldots\ldots\ ?)$

$\therefore\ x^3 - 5x^2 + 7x - 3 = (x - 1)(x^2 \pm ?x + 3)$ want $(-1)(+3) = -3$

$\therefore\ x^3 - 5x^2 + 7x - 3 = x^3 \ldots\ldots + 3x - 1x^2 \ldots\ldots - 3$

$\therefore\ x^3 - 5x^2 + 7x - 3 = x^3 + 4x + 3x - 1x^2 - 4x^2 - 3$

$\therefore\ x^3 - 5x^2 + 7x - 3 = (x - 1)(x^2 - 4x + 3)$

$\therefore\ f(x) = (x - 1)(x^2 - 4x + 3) = (x - 1)(x - 3)(x - 1) = 0$

$\therefore\ \boldsymbol{x = 1}$ of $\boldsymbol{x = 3}$ of $\boldsymbol{x = 1}$

(6) As $f(x) = 2x^3 - 2x^2 - 13x - 9$

<u>x-afsnitte</u>: ($y = 0$) - [Faktorstelling en sintetiese deling – formule.]

$\therefore\ f(1) = 2(1)^3 - 2(1)^2 - 13(1) - 9 = -22$

$\therefore\ (x - 1)$ is nie 'n faktor van $f(x)$ nie.

$\therefore\ f(-1) = 2(-1)^3 - 2(-1)^2 - 13(-1) - 9 = 0$

$\therefore\ (x + 1)$ is 'n faktor van $f(x)$

$$\begin{array}{r|rrrr} & 2 & -2 & -13 & -9 \\ \mathbf{-1} & 0 & -2 & +4 & +9 \\ \hline & 2 & -4 & -9 & +0 \end{array}$$

$\therefore\ (x + 1)(2x^2 - 4x - 9) = 0$

$\therefore\ (x + 1) = 0$ of $(2x^2 - 4x - 9) = 0$

$\therefore\ \boldsymbol{x = -1}$ $\quad\quad$ $\therefore\ a = 2\ ;\ b = -4$ en $c = -9$

$$\therefore\quad x = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-9)\,ac}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 + 72}}{4}$$

$$= \frac{4 \pm \sqrt{88}}{4}$$

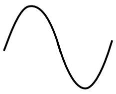
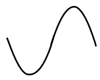
$$x = \frac{4}{4} \pm \frac{2\sqrt{22}}{4}$$

$$\therefore\ \boldsymbol{x = 1 \pm \frac{\sqrt{22}}{2}}$$

A4.2.2 Skets van derdegraadse funksies:

Standaardvorm: $y = ax^3 + bx^2 + cx + d$

Om die grafiek te skets, bepaal:

* **Vorm**: as $a > 0 \rightarrow$  of as $a < 0 \rightarrow$ 

* ***y*-afsnit** $\rightarrow$ $x = 0$ $\therefore$ $(0 ; d)$

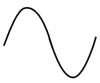
* ***x*-afsnit(te)** $\rightarrow$ $y = 0$ [Sien A4.2.1]

* **Stationêre punte (draaipunte)** $\rightarrow$ $f'(x) = 0$
  Staan ook bekend as die lokale maksimum en die lokale minimum

* **Buigpunt** $\rightarrow$ $f''(x) = 0$

Vb.5 Skets: $f(x) = x^3 + x^2 - x - 1$

****

* **Vorm:** as $a > 0 \rightarrow$ 

* ***y*-afsnit** $\rightarrow$ $x = 0$

  $f(0) = (0)^3 + (0)^2 - (0) - 1 = -1$

  $\therefore$ $(0 ; -1)$

* ***x*-afsnit(te)** $\rightarrow$ $y = 0$

  $\therefore\ 0 = x^3 + 1x^2 - x - 1$

  $\therefore\ 0 = x^2(x + 1) - 1(x + 1)$

  $\therefore\ 0 = (x + 1)(x^2 - 1)$

  $\therefore\ 0 = (x + 1)(x - 1)(x + 1)$

  $\therefore$ $\boldsymbol{x = -1}$ of $\boldsymbol{x = 1}$ of $\boldsymbol{x = -1}$

  $\therefore$ $(-1 ; 0)$; $(1 ; 0)$ en $(-1 ; 0)$ $\rightarrow$ Indien 2 x-afsnitte dieselfde is, sal die grafiek op die x-as draai. $\therefore$ $(-1 ; 0)$ is dan een van die draaipunte.

* **Stationêre punte (draaipunte)** $\rightarrow$ $f'(x) = 0$
  Staan ook bekend as die lokale maksimum en die lokale minimum

$$f(x) = x^3 + x^2 - x - 1$$

$$f'(x) = 3x^2 + 2x - 1$$

$$0 = (x + 1)(3x - 1)$$

x-afsnit (-1 ; 0) $\rightarrow$ draaipunt.

$\therefore x = -1$ of $x = \frac{1}{3}$

$$\therefore f(-1) = (-1)^3 + (-1)^2 - (-1) - 1$$

$$\therefore f(-1) = 0$$

$$\therefore (-1\,;0)$$

$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) - 1$$

$$f\left(\frac{1}{3}\right) = \frac{-32}{27}$$

$$\left(\frac{1}{3}\,;\frac{-32}{27}\right)$$

* **Buigpunt** $\rightarrow f''(x) = 0$

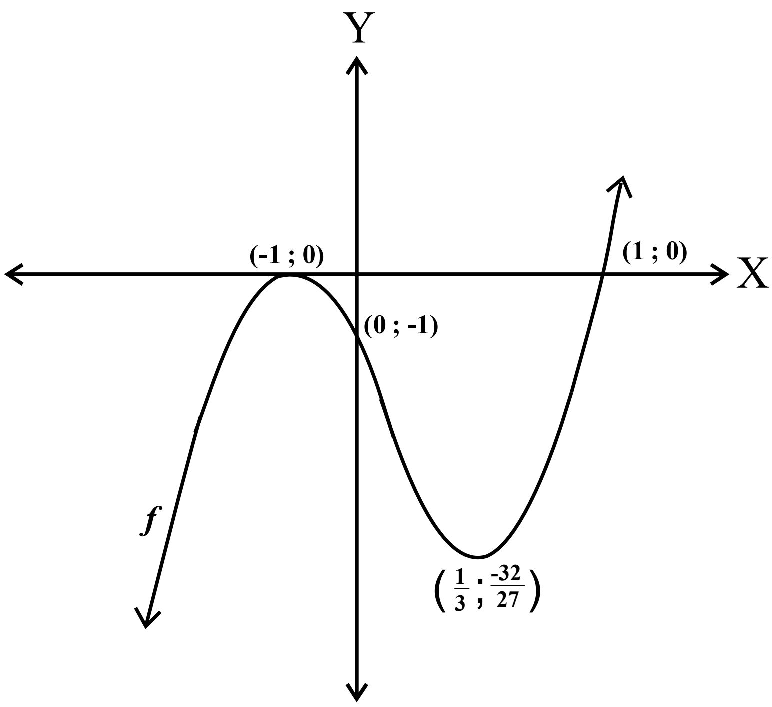
$$f(x) = x^3 + x^2 - x - 1$$

$$f'(x) = 3x^2 + 2x - 1$$

$$f''(x) = 6x + 2 = 0$$

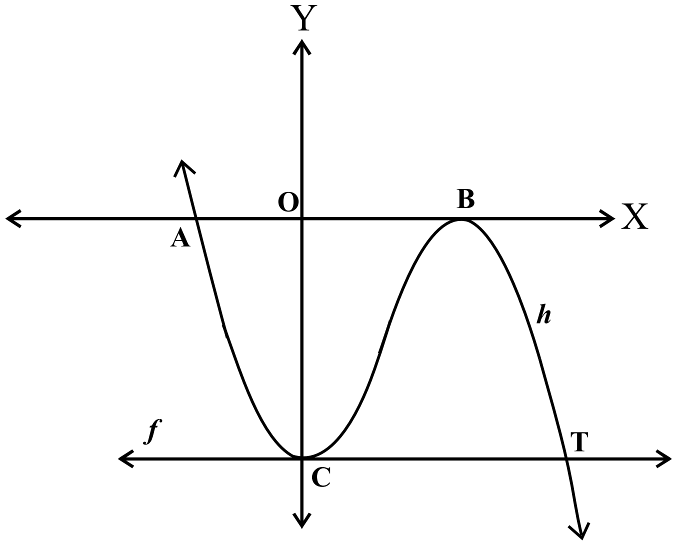
$$\therefore x = \frac{-2}{6} = \frac{-1}{3}$$

SKETS:



A4.2.2 **Bepaal die vergelyking van derdegraadse funksies:**

Vb.6 $h(x) = ax^3 + bx^2 + cx + d$ *met A(-1 ; 0) ; B(2 ; 0) en C(0 ; -8)*
CT is ewewydig aan die x-as.



Bereken: (a) die waardes van a ; b ; c en d.
(b) die vergelyking van f.
(c) die vergelyking van die raaklyn (g) aan kromme h, deur A.
(d) die x-koördinaat van die buigpunt van h.
(e) die koördinate van T.
(f) die gemiddelde gradiënt tussen B en C.
(g) die x-waardes waarvoor die h dalend sal wees.
(h) x as h(x) < 0.

****

(a) $x = -1$ *of* $x = 2$ *of* $x = 2$ *koördinate van A en B*

$\therefore\ h(x) = a(x + 1)(x - 2)(x - 2)$ *kurwe raak by B → 2 herhaal*

$\therefore\ h(x) = a(x + 1)(x^2 - 4x + 4)$

$\therefore\ h(x) = a(x^3 - 4x^2 + 4x + 1x^2 - 4x + 4)$

$\therefore\ h(x) = a(x^3 - 3x^2 + 4)$

x y → y = h(x)

$\therefore\ h(0) = a[(0)^3 - 3(0)^2 + 4]$ *deur C(0 ; -8)*

$\therefore\ -8 = a(4)$

$\therefore\ a = -2$

$\therefore\ h(x) = -2(x^3 - 3x^2 + 4)$

$\therefore\ h(x) = -2x^3 + 6x^2 - 8$

$\therefore$ ***a = -2 ; b = +6 ; c = 0 en d = -8***

(b) $\boldsymbol{f(x) = -8}$ raaklyn aan draaipunt $\rightarrow m = 0$

(c) Vergelyking raaklyn: $y - y_1 = m(x - x_1)$ met $m = h'(x)$

$\therefore \quad h'(x) = -6x^2 + 12x$

$\therefore \quad h'(-1) = -6(-1)^2 + 12(-1)$ raak by A $\rightarrow x = -1$

$\therefore \quad m = -18$

$\therefore \quad y - 0 = -18(x - (-1))$ deur $A(-1\,;\,0)$

$\therefore \quad \boldsymbol{y = -18x - 18}$

(d) Buigpunt $\rightarrow h''(x) = 0$

$h(x) = -2x^3 + 6x^2 - 8$

$\therefore \quad h'(x) = -6x^2 + 12x$

$\therefore \quad h''(x) = -12x + 12 = 0$

$-12x = -12$

$\therefore \boldsymbol{x = 1}$

(e) T is die snypunt van h en f:

$h(x) = -2x^3 + 6x^2 - 8$ en $f(x) = -8 = y \rightarrow$ vervang in h

$\therefore \quad -8 = -2x^3 + 6x^2 - 8$

$\therefore \quad 0 = -2x^3 + 6x^2$

$\therefore \quad 0 = x^3 - 3x^2$

$\therefore \quad 0 = x^2(x - 3)$

$\therefore \quad x = 0$ of $x = 3$

$\therefore \quad \boldsymbol{T(3\,;\,-8)}$

(f) $GG = \dfrac{y_2 - y_1}{x_2 - x_1}$ deur $\overset{x_1 \;\; y_1}{B(2\,;\,0)}$ en $\overset{x_2 \;\; y_2}{C(0\,;\,-8)}$

$\therefore GG = \dfrac{-8 - 0}{0 - 2} = \dfrac{-8}{-2} = \boldsymbol{4}$

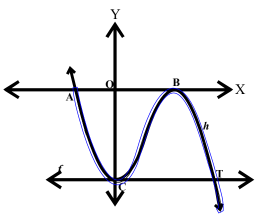
(g) h is dalend as $x \in (-\infty\,;\,\mathbf{0}) \cup (\mathbf{2}\,;\,\infty)$

(h) $h(x) < 0$

$\boldsymbol{x > -1 \text{ maar } x \neq 2}$

$\uparrow$

$h(x) < 0$ en nie $h(x) \leq 0$



A4.3 Verdere toepassings:

- Minimum / maksimum $\rightarrow \quad f'(x) = 0$

- Afstand = spoed $\times$ tyd $\rightarrow$ Spoed / snelheid is die tempo waarteen afstand verander met tydsverloop.
  $\therefore$ As $s(t)$ die afstand verteenwoordig na tydsduur t, sal die snelheid $\rightarrow s'(t)$

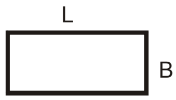
- Versnelling is die tempo waarteen snelheid verander $\rightarrow \therefore \ s''(t)$

- Enige tempo van verandering: $f'(x)$

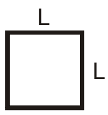
- Maksimum / minimum omtrek, oppervlak of volume $\rightarrow f'(x) = 0$

Hersien formules vir omtrek, oppervlak en volume:

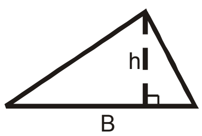
* Reghoek: Omtrek = 2L + 2B
Oppervlakte = L $\times$ B



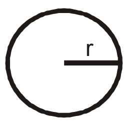
* Vierkant: Omtrek = 4L
Oppervlakte = L^2



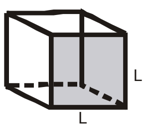
* Driehoek: Omtrek = sy + sy + sy
Oppervlakte = $\frac{1}{2}B \times \perp h$



* Sirkel: Omtrek = $2\pi r$
Oppervlakte = πr^2

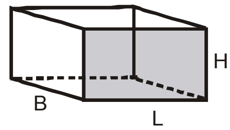


* Kubus: $\therefore$ **Buite - oppervlak area = $6L^2$**
en **Volume = L^3**



* Reghoekige prisma: $\therefore$ **Buite - oppervlak area = 2LH + 2BH + 2LB**

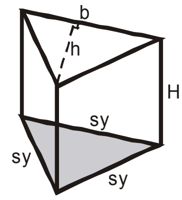
en **Volume = L × B × H**



* Driehoekige prisma: **Buite - oppervlak area = (sy + sy + sy)H + 2 × $\frac{1}{2}$ bh**

$\therefore$ **Buite - oppervlak area = (sy + sy + sy)H + bh**

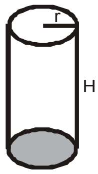
en **Volume = $\frac{1}{2}$bh × H**



* Silinder: **Buite - oppervlak area = $(2\pi r)H + 2(\pi r^2)$**

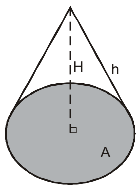
$\therefore$ **Buite - oppervlak area = $2\pi r\,(H + r)$**

en **Volume = V = $\pi r^2 H$**



* Regte kegel:

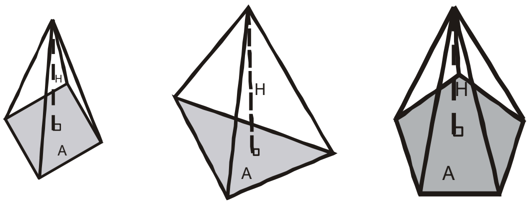
Opp. area = $\pi r(h + r)$ Met: A die opp van die basis– benodig vir die volume!
r die radius van die basis (sirkel)
h die skuinshoogte van die kegel
H die loodregte hoogte – benodig vir die volume!



en **Volume = $\frac{1}{3}$AH**

* Piramide:

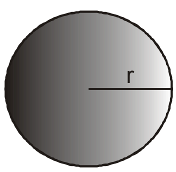
Opp. area = A + $\frac{1}{2}$ph Met: A die oppervlakte van die basis
p die omtrek van die basis
h die skuinshoogte van die piramide
H die loodregte hoogte – benodig vir die volume!

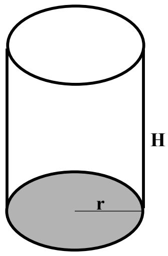


en **Volume = $\frac{1}{3}$AH**

* Sfeer: **Opp. area = $4\pi r^2$ met r die radius.**

en **Volume = $\frac{4}{3}\pi r^3$**





Vb.7 'n Oop silindriese kartonhouer (sien langsaan) word vervaardig.
Die volume van die houer is 2 155 cm^3.

(a) Skryf H in terme van die volume en r.

(b) Toon aan dat die totale buite-oppervlak van die silinder: P(r) = $\pi r^2 + \frac{4\,310}{r}$

(c) Bereken r, korrek tot 2 desimale, waarvoor die buite-oppervlak van die kartonhouer 'n minimum sal wees.

****

(a) $V = \pi r^2 H$

$\therefore\ 2\,155 = \pi r^2 H$

$\therefore\ \boldsymbol{H = \frac{2\,155}{\pi r^2}}$

(b) $BO = 2\pi r^2 + 2\pi r H - \color{red}{\pi r^2}$ *Silinder is oop bo – dus slegs een sirkel onder!*

$\therefore\ P(r) = \pi r^2 + 2\cancel{\pi r}\left(\frac{2\,155}{\cancel{\pi} r^{\cancel{2}}}\right)$ *Vervang H soos in (a) bepaal.*

$= \pi r^2 + 2\left(\frac{2\,155}{r}\right)$

$\therefore\ \boldsymbol{P(r) = \pi r^2 + \frac{4\,310}{r}}$

(c) Minimum $\rightarrow P'(r) = 0$

Maar $P(r) = \pi r^2 + \frac{4\,310}{r} = \pi r^2 + 4\,310 r^{-1}$

$\therefore\ P'(r) = 2\pi r - 4\,130 r^{-2} = 0$

$\therefore\ 2\pi r = \frac{4\,130}{r^2}$

$\therefore\ 2\pi r^3 = 4\,130$

$\therefore\ r^3 = \frac{4\,130}{2\pi}$

$\therefore\ r = \sqrt[3]{657{,}309915}$

$\therefore\ \boldsymbol{r = 8{,}69\ cm}$

Vb.8 *Die tempo waarteen organisme "X" groei word gegee deur:*
$g(t) = -t^2 + 10t + 62$ *met* t *die tydsverloop in minute en die massa van die organisme* g *in gram.*

Bepaal: *(a) organisme "X" se aanvangs massa.*
(b) die massa van organisme "X" na 3 minute.
(c) na hoeveel minute organisme "X" maksimum massa sal bereik.
(d) organisme "X" se maksimum massa.

****

(a) Aanvang $\rightarrow t = 0$

$\therefore\ g(0) = -(0)^2 + 10(0) + 62 = 62$

$\therefore$ *Organisme "X" se aanvangs massa is 62 gram.*

(b) Na 3 minute $\rightarrow t = 3$

$\therefore\ g(3) = -(3)^2 + 10(3) + 62 = 83$

$\therefore$ *Organisme "X" se massa na 3 minute is 83 gram.*

(c) Maksimum massa $\rightarrow g'(t) = 0$

$$g(t) = -t^2 + 10t + 62$$

$$\therefore\ g'(t) = -2t + 10 = 0$$

$$\therefore\ -2t = -10$$

$$\therefore\ t = 5$$

$\therefore$ *Organisme "X" bereik maksimum massa na 5 minute.*

(d) Maksimum massa na 5 minute.

$\therefore\ g(5) = -(5)^2 + 10(5) + 62 = 87$

$\therefore$ *Organisme "X" se maksimum massa is 87 gram.*