Agtergrondkennis – Algebraïese uitdrukkings en vergelykings

C1 Produkte:
C1.1 Distributiewe wet:

Vb. 1 Bepaal die volgende produkte deur gebruik te maak van die distributiewe wet:

(a) $(x - 2)(x + 2)$ (b) $(3m + n)(2m + 5n)$

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(a) $x(x + 2) - 2(x + 2)$

$= x^2 + 2x - 2x - 4$

$= \mathbf{x^2 - 4}$

(b) $(3m + 1n)(2m + 5n)$

$= 6m^2 + 15mn + 2mn + 5n^2$

$= \mathbf{6m^2 + 17mn + 5n^2}$

Vb. 2 Vereenvoudig: (a) $(2a + 1)(2a - 1) = 4a^2 - 2a + 2a - 1 = \mathbf{4a^2 - 1}$

(b) $(m^2 - 5n)(m^2 + 5n) = m^4 + 5m^2n - 5m^2n - 25n^2 = \mathbf{m^4 - 25n^2}$

of korter! (c) $(xy + 3)(xy - 3) = \mathbf{x^2y^2 - 9}$

(d) $\left(\frac{ab}{4} - \frac{1}{7}\right)\left(\frac{ab}{4} + \frac{1}{7}\right) = \mathbf{\frac{a^2b^2}{16} - \frac{1}{49}}$

C1.2 Kwadrering van ′n tweeterm:

Vb. 3 Bepaal die volgende produkte:

(a) $(2x + 1)^2$ (b) $\left(m - \frac{1}{m}\right)^2$

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(a) $(2x + 1)(2x + 1)$

$= 4x^2 + 2x + 2x + 1$

$= \mathbf{4x^2 + 4x + 1}$

(b) $\left(m - \frac{1}{m}\right)\left(m - \frac{1}{m}\right)$

$= m^2 - 1 - 1 + \frac{1}{m^2}$

$= \mathbf{m^2 - 2 + \frac{1}{m^2}}$

Vb. 4 Vereenvoudig: (Korter metode!)

(a) $(m + 5n)^2 = (m)^2 + 2(m)(5n) + (5n)^2 = \mathbf{m^2 + 10mn + 25n^2}$

(b) $(pq - 2)^2 = \mathbf{p^2q^2 - 4pq + 4}$

(c) $\left(\frac{1}{3} + 3x\right)^2 = \mathbf{\frac{1}{9} + 2x + 9x^2}$

C1.3 Tweeterme en drieterme:

Vb. 5 Vereenvoudig die volgende produkte: $(4y + 1)(y^2 - y + 5)$

$= 4y^3 - 4y^2 + 20y + 1y^2 - 1y + 5$

$= \mathbf{4y^3 - 3y^2 + 19y + 5}$

C1.4 Die som en verskil van twee derdemagte:

Vb. 6 Beskou die volgende:

(a) $(x - 3)(x^2 + 3x + 9) = x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 = \mathbf{x^3 - 27}$

(b) $(y + 5)(y^2 - 5y + 25) = y^3 - 5y^2 + 25y + 5y^2 - 25y + 125 = \mathbf{y^3 + 125}$

(c) $(4m - 1)(16m^2 + 4m + 1) = \mathbf{64m^3 - 1}$

(d) $(n^2 + 2)(n^4 - 2n^2 + 4) = \mathbf{n^6 + 8}$

Patroon: Beskou nr (d):

$(n^2)^2 = n^4$ [1ste term 1ste hakie gekwadreer = 1ste term 2de hakie]

$2^2 = 4$ [2de term 1ste hakie gekwadreer = 3de term (altyd +) 2de hakie]

$n^2 \times (+2) = -2n^2$ [1ste term (+) 1ste hakie × 2de term (+) 1ste hakie = - (2de term 2de hakie)]

C1.5 Vereenvoudiging van uitdrukkings:

Volgorde van bewerkings: **(1) Hakies**
(2) Magsverheffing en worteltrekking
(3) Van → ×
(4) Vermenigvuldiging en deling
(5) Plus en minus

Vb. 7 Vereenvoudig die volgende:

(a) $x(x - 2) + 3(x - 2)(x + 2)$

$= x^2 - 2x + 3(x^2 - 4)$

$= x^2 - 2x + 3x^2 - 12$

$= \mathbf{4x^2 - 2x - 12}$

(b) $(m - 4n)^2 - (n + m)^2$

$= m^2 - 8mn + 16n^2 - 1(n^2 + 2mn + m^2)$

$= 1m^2 - 8mn + 16n^2 - 1n^2 - 2mn - 1m^2$

$= \mathbf{15n^2 - 10mn}$

C2 Faktorisering:
C2.1 Gemeenskaplike faktor:

Vb.8 Faktoriseer volledig:

(a) $12mx^2 - 2mx = \mathbf{2mx(6x - 1)}$

(b) $p^3q^2r + p^2qr^3 = \mathbf{p^2qr(pq + r^2)}$

(c) $2(m + n) + y(m + n) = \mathbf{(m + n)(2 + y)}$

(d) $2a(x - y) - 5(y - x) = 2a(x - y) + 5(x - y) = \mathbf{(x - y)(2a + 5)}$

C2.2 Groepering:

Vb. 9 Faktoriseer volledig: (a) $ax + ay + bx + by$

$= \underline{ax + ay} + \underline{bx + by} = a(x + y) + b(x + y)$

$= \mathbf{(x + y)(a + b)}$

(b) $4m^2 - pn + n - 4m^2p$

$= \underline{4m^2 + n} - \underline{pn - 4m^2p} = 1(4m^2 + n) - p(n + 4m^2)$

$= \mathbf{(4m^2 + n)(1 - p)}$

C2.3 Verskil tussen twee vierkante:

Vb. 10 Faktoriseer volledig: (a) $x^2 - y^2 = \mathbf{(x - y)(x + y)}$

(b) $144r^2 - p^{16} = \mathbf{(12r + p^8)(12r - p^8)}$

(c) $m^2n^2 - (2mn + 1)^2 = [mn - (2mn + 1)][mn + (2mn + 1)]$

$= [mn - 2mn - 1][mn + 2mn + 1]$

$= \mathbf{[-mn - 1][3mn + 1]}$

C2.4 Drieterme:

Vb. 11 Faktoriseer volledig: (a) $x^2 + 4x + 3 = \mathbf{(x + 3)(x + 1)}$

(b) $x^2 - 7x + 12 = \mathbf{(x - 3)(x - 4)}$

(c) $y^2 + 6y - 7 = \mathbf{(y + 7)(y - 1)}$

(d) $m^2 - m - 12 = \mathbf{(m - 4)(m + 3)}$

C2.5 <u>Nog drieterme</u>:

Vb. 12 Faktoriseer volledig: (a) $8x^2 + \mathbf{2x} - 15$

$$\begin{array}{ccccc} 4x & & -5 & \longrightarrow & -10x \\ \times & \times & \times & & + \\ 2x & & +3 & \longrightarrow & \underline{+12x} \\ & & & & \mathbf{+\,2x} \end{array}$$

$= \mathbf{(4x - 5)(2x + 3)}$

(b) $5a^2 + \mathbf{17ab} + 6b^2$

$$\begin{array}{ccccc} 5a & & +2b & \longrightarrow & +\,2ab \\ \times & \times & \times & & + \\ 1a & & +3b & \longrightarrow & +\underline{15ab} \\ & & & & \mathbf{17ab} \end{array}$$

$= \mathbf{(5a + 2b)(1a + 3b)}$

C2.6 <u>Som en verskil van derdemagte</u>:

Vb. 13 Faktoriseer volledig: (a) $x^3 - y^3 = \mathbf{(x - y)(x^2 + xy + y^2)}$

(b) $x^3 + y = \mathbf{(x + y)(x^2 - xy + y^2)}$

VIR DIE PATROON SIEN C1.4

C3 <u>Algebraïese breuke</u>:
C3.1 <u>Vermenigvuldiging en deling</u>:

Vb.14 Vereenvoudig (aanvaar dat geen noemer nul is nie!):

(a) $\dfrac{12y - 3y^2}{6y} = \dfrac{{}^{1}\cancel{3}(4 - y)}{{}^{2}\cancel{6}y} = \dfrac{\mathbf{4 - y}}{\mathbf{2}}$

(b) $\dfrac{x^2(1 - y) + x(1 - y) - 6(1 - y)}{(x + 3)(y - 1)}$

$= \dfrac{(1 - y)(x^2 + x - 6)}{(x + 3)(1 - y)}$

$= \dfrac{(\cancel{1 - y})(x^2 + x - 6)}{(x + 3)(\cancel{1 - y})}$

$= \dfrac{-(\cancel{x + 3})(x - 2)}{(\cancel{x + 3})}$

$= \mathbf{-(x - 2)}$

(c) $\dfrac{6ab^2}{5ac} \times 3b^3c^2 \div \dfrac{18a^2bc}{10a} = \dfrac{6ab^2}{5ac} \times \dfrac{3b^3c^2}{1} \times \dfrac{10a}{18a^2bc}$

$$= \frac{180a^2b^5c^2}{90a^3b^1c^2} = \frac{\mathbf{2b^4}}{\mathbf{a}}$$

(d) $\dfrac{y^2 + y}{y^2 + 2y + 1} \div \dfrac{y^3 + y^2 - 2y}{y^2 - 1}$

$$= \frac{y\cancel{(y+1)}}{\cancel{(y+1)}(y+1)} \times \frac{y^2 - 1}{y^3 + y^2 - 2y}$$

$$= \frac{y}{(y+1)} \times \frac{(y-1)(y+1)}{y(y^2 + y - 2)}$$

$$= \frac{y}{\cancel{(y+1)}} \times \frac{\cancel{(y-1)}\cancel{(y+1)}}{y(y+2)\cancel{(y-1)}}$$

$$= \frac{\mathbf{1}}{\mathbf{(y+2)}}$$

C3.2 <u>Optel en aftrek</u>:

Vb.15 Vereenvoudig. Toon alle beperkings aan!

(a) $\dfrac{2m}{m-n} + \dfrac{3m}{m-n} = \dfrac{2m + 3m}{m-n} = \dfrac{\mathbf{5m}}{\mathbf{m-n}}$ *KGV = m - n*

$\therefore$ *Beperkings:* $m - n \neq 0 \rightarrow m \neq n$

(b) $\dfrac{12}{y^2 - 4} - \dfrac{5}{y^2 + 2y}$

$$= \frac{12}{(y-2)(y+2)} - \frac{5}{y(y+2)}$$

KGV = y(y - 2)(y + 2)

$$= \frac{12}{(y-2)(y+2)} \times \frac{y}{y} - \frac{5}{y(y+2)} \times \frac{(y-2)}{(y-2)}$$

Beperkings: * $y \neq 0$

* $y - 2 \neq 0 \therefore y \neq 2$

* $y + 2 \neq 0 \therefore y \neq -2$

$$= \frac{12 \times y}{(y-2)(y+2)y} - \frac{5 \times (y-2)}{y(y+2)(y-2)}$$

$$= \frac{12y - 5(y-2)}{y(y-2)(y+2)}$$

$$= \frac{12y - 5y + 10}{y(y-2)(y+2)}$$

$$= \frac{\mathbf{7y + 10}}{\mathbf{y(y-2)(y+2)}}$$

C4 Vergelykings:
C4.1 Vergelykings met breuke:

*** Die verskil tussen ′n uitdrukking en ′n vergelyking:**

Algebraïese uitdrukking:

$$\frac{x-1}{x^2-4} + \frac{2}{x^2-2x}$$

$$\therefore \frac{x-1}{(x+2)(x-2)} + \frac{2}{x(x-2)}$$

Vergelyking:

$$\frac{x-1}{x^2-4} = \frac{2}{x^2-2x}$$

$$\therefore \frac{x-1}{(x+2)(x-2)} = \frac{2}{x(x-2)}$$

KGV = $x(x+2)(x-2)$ $\quad\therefore$ $x \neq 0$; $x \neq -2$ en $x \neq 2$

$$\therefore \frac{x(x-1)+2(x+2)}{x(x+2)(x-2)}$$

$$\therefore \frac{x^2-x+2x+4}{x(x+2)(x-2)}$$

$$\therefore \frac{\boldsymbol{x^2+x+4}}{\boldsymbol{x(x+2)(x-2)}}$$

$$\therefore \frac{x-1}{\cancel{(x+2)}\cancel{(x-2)}} \times \frac{x\cancel{(x+2)}\cancel{(x-2)}}{1} = \frac{2}{\cancel{x}\cancel{(x-2)}} \times \frac{\cancel{x}(x+2)\cancel{(x-2)}}{1}$$

$$\therefore x(x-1) = 2(x+2)$$

$$\therefore x^2 - x = 2x + 4$$

$$\therefore x^2 - 3x - 4 = 0$$

$$\therefore (x-4)(x+1) = 0$$

$$\therefore \boldsymbol{x = 4} \quad \text{of} \quad \boldsymbol{x = -1}$$

Vb.16 Los op vir x: $\dfrac{2x-3}{x^2+6x+8} - \dfrac{x-4}{x^2+3x-4} = \dfrac{3x+8}{x^2+x-2}$

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$$\frac{2x-3}{(x+2)(x+4)} - \frac{x-4}{(x+4)(x-1)} = \frac{3x+8}{(x+2)(x-1)}$$

KGV = $(x+2)(x+4)(x-1)$ $\quad\therefore$ $x \neq -2$; $x \neq -4$ en $x \neq 1$

$$\frac{(2x-3)}{\cancel{(x+2)}\cancel{(x+4)}} \times \frac{(x+2)(x+4)(x-1)}{1} - \frac{(x-4)}{\cancel{(x+4)}\cancel{(x-1)}} \times \frac{(x+2)\cancel{(x+4)}\cancel{(x-1)}}{1} = \frac{(3x+8)}{\cancel{(x+2)}\cancel{(x-1)}} \times \frac{\cancel{(x+2)}(x+4)\cancel{(x-1)}}{1}$$

$$\therefore (2x-3)(x-1) - (x-4)(x+2) = (3x+8)(x+4)$$

$$2x^2 - 2x - 3x + 3 - (x^2 + 2x - 4x - 8) = 3x^2 + 12x + 8x + 32$$

$$2x^2 - 5x + 3 - x^2 - 2x + 4x + 8 = 3x^2 + 20x + 32$$

$$x^2 - 3x + 11 = 3x^2 + 20x + 32$$

$$0 = 3x^2 + 20x + 32 - x^2 + 3x - 11$$

$$0 = 2x^2 + 23x + 21$$

$$0 = (2x+21)(x+1)$$

$$2x + 21 = 0 \quad \text{of} \quad x + 1 = 0$$

$$\boldsymbol{x = \frac{-21}{2}} \qquad \boldsymbol{x = -1}$$

C4.2 Oplos van vergelykings mbv kwadrering of worteltrekking:

Vb.17 Los op vir x:

(a) $2(x+3)^2 - 8 = 0$ (b) $2\sqrt{x+1} + x - 2 = 0$

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(a)
$$2(x+3)^2 = 8$$
$$(x+3)^2 = \frac{8}{2}$$
$$(x+3)^2 = 4$$
$$x+3 = \pm\sqrt{4}$$
$$x+3 = \pm 2$$
$$x+3 = 2 \quad \text{of} \quad x+3 = -2$$
$$\mathbf{x = -1} \qquad \mathbf{x = -5}$$

(b)
$$2\sqrt{x+1} = 2 - x$$
$$\left(2\sqrt{x+1}\right)^2 = (2-x)^2$$
$$(2)^2\left(\sqrt{x+1}\right)^2 = 4 - 4x + x^2$$
$$4(x+1) = x^2 - 4x + 4$$
$$4x + 4 = x^2 - 4x + 4$$
$$0 = x^2 - 8x$$
$$0 = x(x-8)$$
$$x = 0 \quad \text{of} \quad x = 8$$

TOETS:

$$*LK = 2\sqrt{0+1} + 0 - 2 \qquad RK = 0$$
$$= 2(1) - 2 = 0$$
$$\therefore LK = RK \qquad \therefore \mathbf{x = 0}$$

$$*LK = 2\sqrt{8+1} + 8 - 2 \qquad RK = 0$$
$$= 2(3) - 6 = 0$$
$$\therefore LK = RK \qquad \therefore \mathbf{x = 8}$$

C4.3 K-metode / Substitusie:

Vb.18 Los op vir x: $(x^2 - x)^2 - 8(x^2 - x) + 12 = 0$

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Ons eerste reaksie is gewoonlik om eers die hakies te verwyder:
$x^4 - 2x^3 + x^2 - 8x^2 + 8x + 12 = 0$
Maar dan het ons 'n vierde graadse vergelyking om op te los!
Dit kan ons nog nie doen nie!
Daarom beraam ons 'n ander plan naamlik substitusie of anders bekend as die k-metode:

Vervang elke $(x^2 - x)$ *met 'n k:* $\therefore$ *Stel* $(x^2 - x) = k$

$$\therefore k^2 - 8k + 12 = 0$$
$$\therefore (k-6)(k-2) = 0$$
$$\therefore \mathbf{k} - 6 = 0 \quad \text{of} \quad \mathbf{k} - 2 = 0$$
$$\therefore \mathbf{x^2 - x} - 6 = 0 \quad \text{of} \quad \mathbf{x^2 - x} - 2 = 0 \qquad [\text{vervang } k \text{ weer met } (x^2 - x)]$$
$$\therefore (x-3)(x+2) = 0 \qquad (x-2)(x+1) = 0$$
$$\therefore \mathbf{x = 3} \quad \text{of} \quad \mathbf{x = -2} \qquad \mathbf{x = 2} \quad \text{of} \quad \mathbf{x = -1}$$

C4.4 Vierkantsvoltooiing:

Die volgende is voorbeelde van volkome vierkante:
16 ; x^2 ; $(-y)^2$; $(x - 2)^2$; $(p + 5)^2$ ens.

Netso is $(x^2 - 8x + 16)$ 'n volkome vierkant want
$(x^2 - 8x + 16) = (x - 4)(x - 4) = (x - 4)^2$

∴ Enige uitdukking in die vorm $x^2 + bx$ kan as 'n volkome vierkant geskryf word indien die regte konstante waarde (c) bygetel word: $c = \left(\frac{b}{2}\right)^2$

Bv. $x^2 - 6x \rightarrow x^2 - 6x + \left(\frac{-6}{2}\right)^2 \rightarrow x^2 - 6x + \mathbf{9} = (x - 3)^2 \rightarrow$ volkome vierkant!

of $x^2 + 9x \rightarrow x^2 + 9x + \left(\frac{9}{2}\right)^2 \rightarrow x^2 - 6x + \mathbf{20{,}25} = \left(x + \frac{9}{2}\right)^2 \rightarrow$ volkome vierkant!

Vb.19 Los op vir x deur gebruik te maak van vierkantsvoltooiing:

(a) $x^2 - 4x - 12 = 0$ *(b)* $2x^2 + 3x + 1 = 0$

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(a)

$$x^2 - 4x = 12$$
$$x^2 - 4x + \left(\frac{4}{2}\right)^2 = 12 + \left(\frac{4}{2}\right)^2$$
$$x^2 - 4x + 4 = 12 + 4$$
$$(x - 2)^2 = 16$$
$$\sqrt{(x - 2)^2} = \pm\sqrt{16}$$
$$x - 2 = \pm 4$$
$$\therefore\ x - 2 = 4 \quad \textit{of} \quad x - 2 = -4$$
$$\boldsymbol{x = 6} \qquad \boldsymbol{x = -2}$$

(b)

$$2x^2 + 3x = -1$$
$$\frac{2x^2}{2} + \frac{3x}{2} = \frac{-1}{2}$$
$$x^2 + \frac{3}{2}x = \frac{-1}{2}$$
$$x^2 + \frac{3}{2}x + \left(\frac{3}{2} \div 2\right)^2 = \frac{-1}{2} + \left(\frac{3}{2} \div 2\right)^2$$
$$x^2 + \frac{3}{2}x + \left(\frac{3}{2} \times \frac{1}{2}\right)^2 = \frac{-1}{2} + \left(\frac{3}{2} \times \frac{1}{2}\right)^2$$
$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{-1}{2} + \left(\frac{3}{4}\right)^2$$
$$\left(x + \frac{3}{4}\right)^2 = \frac{-1}{2} + \frac{9}{16}$$
$$\left(x + \frac{3}{4}\right)^2 = \frac{1}{16}$$
$$x + \frac{3}{4} = \pm\frac{1}{4}$$
$$\therefore\ x + \frac{3}{4} = \frac{1}{4} \quad \textit{of} \quad x + \frac{3}{4} = -\frac{1}{4}$$
$$x = -\frac{3}{4} + \frac{1}{4} \qquad x = -\frac{3}{4} - \frac{1}{4}$$
$$\boldsymbol{x = \frac{-2}{4} = \frac{-1}{2}} \qquad \boldsymbol{x = \frac{-4}{4} = -1}$$

C4.5 Kwadratiese formule:

As x opgelos word in $ax^2 + bx + c = 0$ in terme van a, b en c, word die volgende antwoord verkry:

$$x = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2a}$$

Bogenoemde word gesien as die formule wat gebruik word, waarmee enige kwadratiese vergelyking, $ax^2 + bx + c = 0$, opgelos kan word.

Vb.20 Los op vir x: $2(x - 3)(x + 1) = 3$
[Indien nodig, laat die antwoord in eenvoudigste wortelvorm.]

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$$2(x - 3)(x + 1) = 3$$
$$2(x^2 - 2x - 3) = 3$$
$$2x^2 - 4x - 6 - 3 = 0$$
$$2x^2 - 4x - 9 = 0$$

$\therefore\ a = 2 \quad ; \quad b = -4 \quad en \quad c = -9$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-9)\,ac}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 + 72}}{4}$$

$$= \frac{4 \pm \sqrt{88}}{4}$$

$$x = \frac{4}{4} \pm \frac{2\sqrt{22}}{4}$$

$$\therefore\ \boldsymbol{x = 1 \pm \frac{\sqrt{22}}{2}}$$

C5 Aard van wortels:

Indien 'n vergelyking opgelos word, verwys "wortels" na die oplossing (x-waardes) van die vergelyking. Dit word ook die "nulpunte" genoem.

In die kwadratiese formule $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ sal die aard van die wortels bepaal word deur die "$b^2 - 4ac$". Dit word voorgestel as $\Delta = b^2 - 4ac$ met "Δ" genoem die diskriminant of delta.

Die aard van die wortels verwys na of daar wortels bestaan of nie en indien wel of die wortels gelyk of ongelyk ens. sal wees.

Opsomming van die aard:

- Indien $\Delta < 0$, dan is die wortels **nie-reeël**. ∴ Daar is geen x-afsnitte nie.
- Indien $\Delta \geq 0$, dan is die wortels **reeël**. ∴ Daar is x-afsnitte.
  - Indien $\Delta = 0$, dan is die wortels **gelyk**. ∴ Die x-afsnitte is bv. $x = 1$ of $x = 1$.
  - Indien $\Delta > 0$, dan is die wortels **ongelyk**. ∴ Die x-afsnitte is bv. $x = 3$ of $x = -1$.
    - Indien Δ 'n volkome vierkant is, dan is die wortels **rasionaal**.
      ∴ Die x-afsnitte is bv. $x = 0$ of $x = 4\frac{1}{2}$. (Kan verkry word deur faktorisering.)
    - Indien Δ nie 'n volkome vierkant is nie, dan is die wortels **irrasionaal**.
      ∴ Die x-afsnitte is bv. $x = \sqrt{2}$ of $x = -3{,}4763$…... (Faktorisering nie moontlik; verkry wortels dmv die formule of vierkantsvoltooiing.)

Vb.21 Bepaal die aard van die wortels van: *(a)* $x^2 - 6x + 12 = 0$

(b) $x(x + 6) - 5 = 3x$

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(a) $x^2 - 6x + 12 = 0$

$\therefore a = 1,\ b = -6$ *en* $c = 12$

$\therefore \Delta = b^2 - 4ac$

$= (-6)^2 - 4(1)(12)$

$= 36 - 48$

$\therefore \Delta = -12$

∴ *Die wortels is nie-reëel want* $\Delta < 0$

(b) $x(x + 6) - 5 = 3x$

$x^2 + 6x - 5 - 3x = 0$

$x^2 + 3x - 5 = 0$

$\therefore a = 1,\ b = 3$ *en* $c = -5$

$\therefore \Delta = b^2 - 4ac$

$= (3)^2 - 4(1)(-5)$

$= 9 + 20$

$\therefore \Delta = 29$

∴ *Die wortels is reëel en ongelyk want* $\Delta > 0$ *en verder is die wortels irrasionaal, want 29 is nie 'n volkome vierkant nie.*
Dit beteken dat as die vergelyking opgelos moet word sal die uitdrukking nie gefaktoriseer kan word nie en die wortels moet dus met die formule of deur middel van vierkantsvoltooiing bepaal word.

C6 Kwadratiese ongelykhede:

Metode 1:

Vb.22 Los op vir x: $(x - 3)(x + 2) \leq 0$

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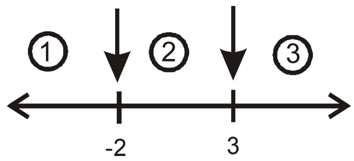


Stel $y = (x - 3)(x + 2)$

Ondersoek die moontlike uitkomste in die verskillende gebiede deur telkens 'n moontlike waarde te kies vir x en dan te bepaal wat die uitkoms sal wees:

Kies enige waarde vir x in gebied 1: $y = (-4 - 3)(-4 + 2) = (-7)(-2) = +14$

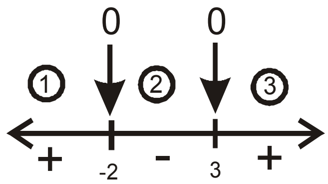
Kies $x = -2$: $y = (-2 - 3)(-2 + 2) = (-5)(0) = 0$

Kies enige waarde vir x in gebied 2: $y = (0 - 3)(0 + 2) = (-3)(2) = -6$

Kies $x = 3$: $y = (3 - 3)(3 + 2) = (0)(5) = 0$

Kies enige waarde vir x in gebied 3: $y = (7 - 3)(7 + 2) = (4)(9) = +36$

$\therefore$ *As* $(x - 3)(x + 2) \leq 0$*, dan lees ons die oplossing af by die negatiewe gebied, maar ook waar die oplossing gelyk is aan 0:*



$\therefore \ \mathbf{-2 \leq x \leq 3}$

Metode 2:

Vb.23 Los op vir x: $x^2 > 4$

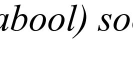
****

$x^2 - 4 > 0$ *[Kry eers 0 aan die een kant.]*

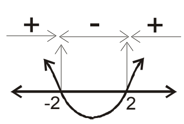
$(x - 2)(x + 2) > 0$ *[Faktoriseer.]*

Nou maak ons gebruik van die kwadratiese funksie (parabool) soos reeds in graad 10 geskets!

Onthou:

As $a > 0$:  *en as* $a < 0$: 

$\therefore$ *Maak 'n rowwe skets van* $y = x^2 - 4$*.*
Dui die vorm en x-afsnitte aan.



$\therefore$ *Oplossing:* $\mathbf{x < -2}$ ***of*** $\mathbf{x > 2}$

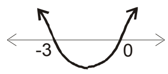
Vb.24 Los op vir x: $-x^2 - 3x \leq 0$

****

Deel deurgaans met -1: $x^2 + 3x \geq 0$

Faktoriseer: $x(x + 3) \geq 0$

Interpreteer oplossing grafies:



Oplossing: $\boldsymbol{x \leq -3}$ ***of*** $\boldsymbol{x \geq 0}$

Vb.25 Los op vir m: $2m^2 + 2 > -4m$

****

$\therefore \quad 2m^2 + 4m + 2 > 0$ *[Kry alle terme aan die een kant]*

$\therefore \quad m^2 + 2m + 1 > 0$ *[Deel deurgaans met 2]*

$\therefore \quad (m + 1)(m + 1) > 0$ *[Faktoriseer]*

$\therefore \quad (m + 1)^2 > 0$

maar enige volkome vierkant, $(\ )^2$, *is altyd positief!*

$\therefore$ $\boldsymbol{m \in \mathbb{R}}$**, *maar*** $\boldsymbol{m \neq -1}$ *[*$(m + 1)^2 \neq 0$*, want daar is net >]*

C7 <u>Gelyktydige vergelykings</u>:

Vb.26 Los op vir x en y: $2x - y = 1$ *en* $2x^2 = 3 - y^2$

****

Uit: $2x - y = 1$

$2x - 1 = y \rightarrow$ *vervang in*

$2x^2 = 3 - y^2$

$2x^2 = 3 - (2x - 1)^2$

$2x^2 = 3 - (4x^2 - 4x + 1)$

$2x^2 = 3 - 4x^2 + 4x - 1$

$6x^2 - 4x - 2 = 0$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$3x + 1 = 0$ *of* $x - 1 = 0$

$\therefore \quad \boldsymbol{x = \frac{-1}{3}} \qquad \boldsymbol{x = 1}$

$2\left(\frac{-1}{3}\right) - 1 = y$

$\frac{-2}{3} - 1 = y$

$\therefore \quad \boldsymbol{y = -1\tfrac{1}{3}}$

$2(1) - 1 = y$

$\therefore \quad \boldsymbol{y = 1}$

$\therefore$ ***Oplossing:*** $\boldsymbol{\left(\frac{-1}{3}\,;1\right)}$ ***of*** $\boldsymbol{\left(1\,;-1\tfrac{1}{3}\right)}$

C8 Eksponensiale vergelykings:

Vb.27 Los vir x op:

(a) $2^x = 16$

(b) $7^{x+1} = 1$

(c) $2.3^x = \frac{2}{9}$

(d) $4^{x-2} \times 8^x = 2^{x(x+1)}$

(e) $3x^{\frac{3}{4}} = 24$

(f) $2^{x-1} + 3.2^{x+1} = 52$

****

(a) $2^x = 16$

$\therefore 2^x = 2^4$

GG $\Leftrightarrow$ *GE* [Gelyke grondtalle $\Leftrightarrow$ Gelyke eksponente]

$\therefore \boldsymbol{x = 4}$

(b) $7^{x+1} = 1$

$\therefore 7^{x+1} = 7^0$

$\therefore x + 1 = 0$

$\therefore \boldsymbol{x = -1}$

(c) $2.3^x = \frac{2}{9}$

$\therefore \frac{\cancel{2} \cdot 3^x}{1} \times \frac{1}{\cancel{2}} = \frac{\cancel{2}}{9} \times \frac{1}{\cancel{2}}$

$\therefore 3^x = \frac{1}{9}$

$\therefore 3^x = \frac{1}{3^2}$

$\therefore 3^x = 3^{-2}$

GG $\Leftrightarrow$ *GE*

$\therefore \boldsymbol{x = -2}$

(d) $4^{x-2} \times 8^x = 2^{x(x+1)}$

$\therefore (2^2)^{x-2} \times (2^3)^x = 2^{x(x+1)}$

$\therefore 2^{2x-4} \times 2^{3x} = 2^{x^2+x}$

$\therefore 2^{2x-4+3x} = 2^{x^2+x}$

$\therefore 2^{5x-4} = 2^{x^2+x}$

GG $\Leftrightarrow$ *GE*

$5x - 4 = x^2 + x$

$0 = x^2 + x - 5x + 4$

$0 = x^2 - 4x + 4$

$0 = (x - 2)(x - 2)$

$\therefore x - 2 = 0 \;\; of \;\; x - 2 = 0$

$\therefore \boldsymbol{x = 2}$

(e)

$$3x^{\frac{3}{4}} = 24$$

$$x^{\frac{3}{4}} = \frac{24}{3}$$

$$x^{\frac{3}{4}} = 8$$

$$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = (2^3)^{\frac{4}{3}}$$

$$\left(x^{\frac{\cancel{3}}{\cancel{4}}}\right)^{\frac{\cancel{4}}{\cancel{3}}} = (2^{\cancel{3}})^{\frac{4}{\cancel{3}}}$$

$$x = 2^4$$

$$\boldsymbol{x = 16}$$

(f)

$$2^{x-1} + 3.2^{x+1} = 52$$

$$2^x.2^{-1} + 3.2^x.2^1 = 52$$

$$2^x(2^{-1} + 3.2) = 52$$

$$2^x\left(\frac{1}{2} + 6\right) = 52$$

$$2^x\left(\frac{13}{2}\right) = 52$$

$$2^x\left(\frac{\cancel{13}}{\cancel{2}}\right) \times \frac{\cancel{2}}{\cancel{13}} = \frac{\cancel{52}^4}{1} \times \frac{2}{\cancel{13}}$$

$$2^x = 2^3$$

$$GG \Leftrightarrow GE$$

$$\therefore \boldsymbol{x = 3}$$